Building Compressed Causal Models of the World

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Abstract

A given causal system can be represented in a variety of ways. How do agents determine which variables to include in their causal representations, and at what level of granularity? Using techniques from Bayesian networks, information theory, and decision theory, we develop a formal theory according to which causal representations reflect a trade-off between compression and informativeness, where the optimal trade-off depends on the decision-theoretic value of information for a given agent in a given context. This theory predicts that, all else being equal, agents prefer causal models that are as compressed as possible. When compression is associated with information loss, however, all else is not equal, and our theory predicts that agents will favor compressed models only when the information they sacrifice is not informative with respect to the agent's anticipated decisions. We then show, across five studies reported here (*N*=1,964) and three studies reported in the supplemental materials (N=789), that participants' preferences over causal models are in keeping with the predictions of our theory. Our theory offers a unification of different dimensions of causal evaluation identified within the philosophy of science (proportionality and stability), and contributes to a more general picture of human cognition according to which the capacity to create compressed (causal) representations plays a central role.

Keywords: Bayesian networks, compression, causal models, proportionality, stability, value of information.

Introduction

Scientists often aim to produce causal models of the world that balance informativeness with compression. That is, they aim to model data-generating processes in a way that captures as much information about those processes as possible, while omitting cumbersome or unnecessary details. For example, epidemiologists might produce a model of cancer rates in a population that treats smoking as a binary variable representing whether or not a person smokes cigarettes, but without specifying the average number of cigarettes the person smokes per day, and omitting additional background variables such as the person's blood type. Ordinary agents face an analogous challenge: in representing the social and physical world around us, each of us must determine which variables to include in our causal models, and at what level of granularity. For example, a causal model of a toddler's tantrums could include whether they napped or not as a binary variable, or a finer-grained specification of the number of minutes they napped; it could include the time of day, or omit this variable entirely. Any such choice of variables instantiates a particular trade-off between informativeness and compression. How do people navigate this choice in building causal models of the world?

In this paper, we begin from the premise that ordinary agents, like scientists, build causal models of the world, and that these causal models can be represented formally as *Bayesian networks* (Gopnik and Tenenbaum, 2007; Griffiths et al., 2008; Pearl, 2000; Spirtes et al., 2000). We then argue that these ordinary agents, like scientists, face a crucial problem: choosing which variables to include in their causal models. Using the Bayes net formalism and a decision-theoretic framework, we provide a formal theory of how to choose variables for a causal model so as to achieve an optimal compression of the environment. We then corroborate our formal framework over the course of five experiments.

A Bayes Net Perspective on Variable Choice

In a Bayesian network, types of events are represented by *random variables*. These random variables are then related to each other by functions that represent the causal relationships between types of events. For instance, in our epidemiological example, a binary variable representing whether or not a person gets cancer will have its value determined by a function that takes as one of its arguments the value of a binary variable denoting whether or not someone smokes.

By definition, each random variable in a Bayesian network can also be represented as a function defined on a set representing all of the possible states of the network's target system. We typically assume that these functions are many-to-one (i.e., that they are surjective but not injective), such that random variables can be understood as compressions (often, massive compressions) of possibility space. For the same data-generating process, there are many different sets of variables that we can define on possibility space, and each choice of variables leads to a different causal model.[1] For example, one model could use a binary variable for a person's smoker status; another could include a variable representing the total number of cigarettes smoked in a person's life. In this case the former model is more compressed, but the latter is more informative. However, the more informative model is not always the superior model. In some cases, it may be no better-supported by the data than more compressed models, or the additional information that it encodes could be irrelevant in a given context. For instance, an epidemiologist will likely be uninterested in whether smokers are more likely to smoke with their right or left hand. Given these considerations, which model should be used in a given modelling context?

[1] Note that we do not assume here that there is a unique, most fine-grained representation of a given data-generating process. Rather, we take the fundamental nature of a data-generating process to be represented by a probability space, with respect to which the random variables in a causal model are measurable. See Appendix A for more details on how the variables in a causal model are defined with respect to a probability space.

In philosophy of science, the question of how to make choices about which variable set to use when representing some system has been termed the “variable choice problem” (Woodward, 2016b). More precisely, the variable choice problem is the problem of determining which normative standards allow us to distinguish between appropriate choices of variables and cumbersome, unnatural choices of variables in contexts where empirical adequacy does not on its own vitiate in favor of one variable set or another. In this paper, we are especially concerned with versions of the variable choice problem that introduce a trade-off between compression and informativeness. As we discuss in the next section, this is a trade-off that arises across a number of areas in cognition and beyond.

The Case for Compression

The importance of compression for understanding one’s environment is well-established within cognitive science. Rosch (1978) argues that classifying types of objects or events in one’s environment requires one to balance the need for informative classification (i.e., maximal informativeness, which introduces pressure towards fine-grained categories) against a need for “cognitive economy” (i.e., less demanding representations, which introduces a pressure towards coarser representations) (see also Chater and Vitányi, 2003). This pressure towards cognitive economy in classification can be understood as a pressure towards compressed representations of one’s environment, where this includes causal representations (see also Fauconnier and Turner, 2008; Murphy, 2004). Keil (2006) argues further that understanding how agents achieve optimal levels of compression in causal representations of their environment is crucial for understanding how those agents explain observed events. For example, if an agent knows that failing to discard old food attracts pests, then that agent can exploit their knowledge of this causal relationship to explain the presence of pests when it occurs, and to intervene on their environment to avoid attracting pests in the future. This agent does not need to separately store a complex causal model of the relationship between different types of food (e.g., grains, vegetables, or meat) and the presence or absence of pests. Thus, compression allows for the recognition of high-level patterns of causal dependence; this accounts for the central role that compression plays in more general cognitive processes such as sense-making and understanding (see also Kirfel et al., 2021; Marzen and DeDeo, 2017; Pacer and Lombrozo, 2017; Wilkenfeld, 2019; Wojtowicz et al., 2021). Finally, Waldmann and Hagmayer (2006) show that people not only group objects together when they infer that they have similar causal effects, but also assume that objects have similar causal effects once they have been grouped together (see also Buchsbaum et al., 2015; Gopnik and Sobel, 2000).

In sum, there is a psychological and philosophical consensus that compression and causal explanation are closely related, and that our choice of variables for causal models often involves a trade-off between informativeness and compression. However, what is missing from this discussion is a precise quantification of the trade-off between informativeness and compression in Bayesian networks, as well as a systematic treatment of the value of the information contained in a Bayesian network for a given agent in a given context.

Outline of a Theoretical Framework and Empirical Hypotheses

Here, we provide a formal theory that quantifies the trade-off between compression and information loss that is often inherent in choices between variable sets in causal modeling. We then test this foundational framework empirically. Next, we show how this trade-off can additionally incorporate considerations about the value of information, reflecting the fact that not all information is equally valuable for a particular agent facing particular decisions. This allows for more compressed causal models that, though they sacrifice informativeness relative to some more detailed model, are nevertheless optimal for a given agent in a given context, because the information lost in compression has no decision-theoretic value for the agent who uses the model to explain and intervene upon their environment. Having developed this more detailed formal framework, we also test it experimentally.

Importantly, on the picture of causal cognition that we use in this paper, we do not presuppose that agents generate a single, complete causal representation of their environment from raw data. Rather, we take it that agents approach their environment with a fragmented and revisable picture of the causal structure of the world. When asked to engage in explicit causal reasoning (e.g., when asked to evaluate the quality of a causal claim, or to generate a causal explanation or description based on data), agents refine this fragmented causal structure and make salient a specific, better-defined causal representation of their environment.

Based on our formal framework, and with this picture of flexible causal representation in the background, we obtain evidence consistent with the following three hypotheses:

H1: In general, people treat compression as a positive feature of a causal representation; all else being equal, the more compressed a given representation is, the better.

H2. Compression can come at the cost of informativeness, and so, all else being equal, the optimal causal representation will achieve a balance of compression and informativeness.

H3: When people are asked to select a causal representation in the context of a particular decision problem, their tolerance for information loss in achieving a more compressed causal claim is moderated by the decision-theoretic value of the information that is lost. That is, when the information that is lost in moving to a more compressed causal claim is not decision-relevant, that information can be sacrificed without sacrificing the overall quality of a causal representation.

The remainder of the paper proceeds as follows. First, we review prior work (primarily from philosophy) on two questions about variable choice with implications for the compression of causal claims. These questions concern the granularity of selected variables (this is reflected in a claim's so-called "proportionality") and the choice of which variables to include (this is reflected in a claim's so-called "stability"). We then introduce a way to formalize both proportionality and stability in terms of the amount of information lost in the move from one causal model to another. This framework allows us to test our first two hypotheses (H1 and H2) empirically, which we do in Experiments 1-2. We also demonstrate how the results of Experiment 2 speak *against* an alternative interpretation of our results in terms of the causal power theory adumbrated in Cheng (1997). In Experiment 3, we rule out an alternative interpretation, showing that a causal contrast theory due to Lien and Cheng (2000) does not predict results that our formal theory is able to successfully accommodate.

Next, we consider how trade-offs between informativeness and compression are moderated by an agent's interests. In particular, information loss could be irrelevant to an agent in a given environment if the lost information would not affect their decisions. Addressing such cases requires extending our formal framework to incorporate the decision-theoretic value of information. After introducing the relevant theory and formalization, we again turn to human judgments to test our third hypothesis (H3) in Experiments 4-5. Specifically, we test the effect of the value of information on the trade-off between informativeness and compression in the evaluation (Experiment 4) and representation (Experiment 5) of causal claims.

Taken together, the paper makes the following novel contributions. On a theoretical level, our framework offers a novel, formal measure of the information lost in moving from one causal model to another, whether that loss is realized through changes in proportionality or stability. In so doing, our framework offers the first unified account of two dimensions of causal evaluation (proportionality and stability) that have previously received independent treatment. Our formalization also allows us to incorporate the decision-theoretic value of information, resulting in a new solution to the variable selection problem in building causal models. Empirically, we offer novel evidence that humans trade off informativeness and compression in evaluating causal claims, and moreover that the evaluation and production of causal claims is sensitive to the decision-theoretic value of information. We find no evidence that these trade-offs are handled differently when compression is achieved through changes in proportionality versus stability, lending empirical support to our formal unification. Finally, our findings contribute to a more general picture of human cognition as balancing tradeoffs between informativeness and compression in context-sensitive ways that take into account the value of information for a given agent in a given context.

Dimensions for Comparing Causal Claims Across Compressions

On its own, the Bayesian network approach to causal representation treats causation as a binary relation; two variables are either causally related or they are not, and so the corresponding causal claim (e.g., "smoking causes cancer") is either appropriate or it is not. However, we take it that one can nevertheless make graded distinctions between causal claims along a large number of different dimensions, in keeping with experimental work on causal judgement (e.g., Cheng, 1997; Gerstenberg et al., 2021; Icard et al., 2017; Lombrozo, 2007, 2010; Morris et al., 2018; O'Neill et al., 2021, 2022; Quillien, 2020; Spellman, 1997). In our framework, we consider two dimensions along which causal models can vary, corresponding to the following two questions about variable choice in building a causal model: i) At what level of granularity should a variable be defined?, and ii) Which variables should be included in a causal model?

These two questions correspond to two graded dimensions that have been discussed in the philosophy literature on causation, especially by Woodward (2008, 2010, 2016, 2021a, 2021b), though by others as well (e.g., Bourrat, 2021; DiMarco, 2021; Franklin-Hall, 2016; Gebharter and Eronen, 2021; Harbecke, 2021; Hoffmann-Kolss, 2014; List and Menzies, 2009; Ross, 2015; Weslake, 2013). These dimensions are *proportionality* and *stability*. A causal claim's 'proportionality' depends on the extent to which it is informative about how possible changes to the cause would result in changes in its effect. Since coarse-graining or refining variables that stand in causal relationships to one another can change the proportionality of causal claims, the concept of proportionality offers a partial answer to the question, "at what level of granularity should a variable be represented?" By contrast, 'stability' refers to the degree to which a causal relationship is insensitive to changes in the values of unspecified background variables, and thus offers a partial answer to the question, "which variables should be included in a causal model?" Below, we describe both proportionality and stability in greater detail.

Proportionality

Proportionality is described by Woodward as the degree to which a causal claim of the form '*C* causes *E*,' where *C* and *E* are variables in a causal structure, is stated at the "level [of causal description] that is most informative about the conditions under which the effect will and will not occur'' (2021a, p. 389). For Woodward, the hierarchy of levels of description with which a causal relationship can be stated corresponds to a sequence of "vertically" related causal variables, where each causal variable in the sequence is a coarsening of the previous causal variables (2021a, p. 371). The standard example of a hierarchy of levels of description that differ with respect to their proportionality comes from Yablo (1992). Consider a pigeon who has been trained to peck at all and only red targets. In a causal model of a system containing this pigeon and various targets, one might have a variable C with the range of values {red target, non-red target}, and another variable E with the range of values {pigeon pecks, pigeon does not peck}. To be an accurate representation of the underlying data-generating structure, the model would have to be such that C is a cause of E. However, one could also generate a causal model in which C is replaced with a variable C' with the range of values {scarlet target, non-scarlet red target, non-red target}. Here, accuracy would also demand that we say that $C'$is a cause of E, since some changes in the value of C' lead to changes in the value of E; changing C' from either of its first two values to its third, and vice versa, leads to changes in the value of E.

According to Woodward's definition, the claim 'C causes E' has the same level of proportionality as the claim 'C'causes E.' This is because a function specifying how changes in C bring about changes in E would give an agent all the information that they need to appropriately manipulate the effect, as would a function specifying how changes in C′ bring about changes in E. However, 'C causes E' achieves a more compressed representation of the data-generating process than 'C'causes E.' This is because C is a *coarsening* of C'. That is, it defines a strictly more general equivalence class on possibility space: any scarlet target or non-scarlet red target is still a red target. So, to the extent that we aim to optimize proportionality in our causal representations, we are *licensed* to use compressed representations when those compressions do not result in a reduction in proportionality, as argued in Woodward (2021b).[2] If the pigeon had instead been trained to peck at scarlet targets (and not other red targets), then the claim 'C' causes E' would be more

[2] In Yablo's original paper, as well as in earlier work on this topic by Woodward (2010), the definition of proportionality was such that the variable with the range of values {red target, non-red target} would be said to be a *more* proportional cause of the variable with range of values {pigeon pecks, pigeon does not peck} than the variable with range of values {scarlet target, non-scarlet red target, non-red target}. However, as Woodward (2021a) notes, the amount of information that the value of either of these causal variables conveys about the value of the effect variable is equal; specifying the value of either causal variable tells us what the value of the effect variable would be. For this reason, we follow the later Woodward in treating the two variables as equally proportional, with this licensing the choice of the more coarse-grained variable.

proportional than 'C causes E,' because the former would be more informative about the conditions under which the effect will and will not occur (i.e., that it will occur for red targets that are scarlet, but not for red targets that are not scarlet).

When testing the influence of proportionality in people's evaluations of causal claims, it makes sense to consider causal models in which compression is achieved by coarsening the range of a particular causal variable. Consider a causal claim $C' \rightarrow E$ that is embedded within a given causal model. If we replace C' with a coarsening C, but leave the rest of the model unchanged, then we compress the model by replacing a variable with its coarsening. We can then assess how much information about the likely value of E is conveyed by changes in C in this more compressed model, as compared to how much information about likely values of E is conveyed by changes in C' in the less compressed model. This tells us how proportional the claim $C \rightarrow E$ is, as compared to the claim $C' \rightarrow E$. Thus, evaluations of the relative proportionality of causal claims involve comparisons of more and less compressed causal models of the same data-generating process. In what follows, this idea will be made mathematically precise.

To date, little empirical work has considered proportionality as a relevant dimension in people's evaluations of causal claims. Of most direct relevance, Lien and Cheng (2000) offer evidence that agents prefer proportional causal claims. Specifically, they show that people prefer to give causal explanations that "explain as much as possible with as few causal rules as possible," (p. 88). By "causal rules," they mean mappings from values of a causal variable to values of an effect variable. This preference for less informationally detailed causal relations tracks a preference for more coarse-grained explanations in cases where fine-graining only serves to complicate the description of the relation between cause and effect. Thus, they find that agents prefer just those kinds of coarsenings that are licensed by a preference for more proportional causal claims. However, their work does not address the question of whether and how judgments of more or less proportional causal claims instantiate a trade-off between compression and informativeness. In other relevant work, Bechlivanidis et al. (2017) report a preference for concreteness over abstraction in causal explanation (i.e., people favor explanations with finer-grained variable choices), even in cases where such concreteness does not add any information that would be relevant for predicting the effect in question. While this suggests that people may ignore compression in favor of greater detail under some conditions, their task involved evaluating causal explanations for token events, not type-level causal claims (see also Aronowitz and Lombrozo, 2020, for potentially relevant discussion). We take explaining why a particular event happened to be a cognitively distinct task from identifying patterns in the causal relations between types of events, such that the norms governing the former might be different from those governing the latter.[3] Our focus here is on the latter cognitive task.

Stability

Stability is the extent to which a causal claim is sensitive to changes in unspecified background conditions (Woodward, 2010).[4] As an example of a highly stable causal relationship, consider "smoking causes lung cancer." Across a wide range of plausible changes to other features of the world, people who smoke are more likely to get lung cancer than those who do not, and this statistical association is due to a causal relationship between smoking and lung cancer. By contrast,

[3] This is not to say that these tasks are entirely unrelated. For example, seeking an explanation for a particular event might lead someone to identify patterns in the causal relations between types of events, and beliefs about such causal relations surely constrain explanations for particular events. Our point here is simply that finding a preference for more detailed descriptions in some explanations for particular events does not imply a preference for finer-grained specifications of type-level causal relationships.

[4] By "unspecified" background conditions, we mean those conditions that are not explicitly part of the causal relationship. E.g., the stability of the claim "smoking causes lung cancer" depends on its persistence across changes in whether a person also lives in a high-air-pollution environment, but the stability of the claim "smoking and living in a high-air pollution environment causes lung cancer" does not, since in the latter case the amount of air pollution in the environment is specified in the causal claim.

consider Woodward's example of a binary variable *C* representing whether or not a person has a particular genetic mutation that typically results in dyslexia, and another binary variable *E* denoting whether or not that person eventually learns to read. Suppose that there is widespread failure to address and correct early reading failures in dyslexic children. Under these conditions, the causal claim '*C* causes *E*' would hold; the presence or absence of the genetic mutation would lead to changes in the likelihood of a person learning to read. However, this claim is highly *un*stable. If we alter the background conditions so that dyslexia is treatable and is treated (as, in fact, it often is), then the causal effect of the genetic mutation on the probability of one's learning to read is greatly diminished.

When we compress a causal model by removing a set of variables, we can regard the removed variables as background conditions, and assess the stability of the causal relationships that remain after compression by observing how well they are preserved even as background conditions are removed. Whether a variable is designated as a background condition as opposed to a causal variable of interest is ultimately a distinction that we impose on the model, rather than one that is dictated by the nature of the system being modelled. That is, we can stipulate that a particular relationship $C \rightarrow E$ in a causal model is of interest, and designate other variables in the model as background conditions.[5] We can then assess the stability of the relationship $C \rightarrow E$ by compressing the model in which it is embedded so as to remove the background conditions, and then measuring the degree to which changes in the value of *C* still result in changes in the probability distribution over *E*. Thus, as in the case of proportionality, we can measure the stability of a causal claim by comparing how much information about the likely value of the effect variable

[5] See Watson and Silva (2022) for an example from the machine learning literature in which background and foreground causal variables are distinguished by stipulation, with fruitful results.

is conveyed by changes in the causal variable when the claim is embedded in a more or less compressed causal model. This too will be made more mathematically precise in what follows.

Prior work offers some direct evidence that people favor causal claims that are more stable over those that are less stable. Specifically, Vasilyeva et al. (2018) provide evidence that people prefer more stable causal claims even when other dimensions of causal variation (such as causal strength) are carefully controlled. As with proportionality, however, this empirical work does not address the question of whether and how judgments of more or less stable causal claims instantiate a trade-off between compression and informativeness.

In sum, both proportionality and stability can be assessed by comparing how informative changes in a causal variable are with respect to likely values of an effect variable across more or less compressed causal models in which the cause-effect relationship is embedded. This suggests that proportionality and stability are actually two species of the same genus. That is, both proportionality and stability are measures of how much information a causal relationship preserves across different types of compression. To test this hypothesis, in our experiments we manipulate whether participants are put in scenarios in which a compression of the implicit causal model involves coarsening a causal variable or eliding a background condition. Across all five experiments, we find no evidence that participants' evaluations and generations of causal claims are affected by whether compression involves coarsening a causal variable or eliminating a background condition. Thus, our unified analysis of proportionality and stability is in keeping with the findings of our experiments, but represents an important departure from prior work within philosophy, which has sometimes aimed to offer a formalization of stability and/or proportionality (e.g., Pocheville et al., 2017), but without offering a unifying framework for both.

Formalizing Information Loss Due to Compression

In this section, we formalize our notion of information loss due to compression, and how it can be used to evaluate both the proportionality and the stability of causal claims. (Our formalization here is given at a relatively high level of abstraction; see Appendix A for a more complete presentation in the language of graphical causal models.) As a preliminary point, note that both a set of causal variables $\boldsymbol{C}$ and an effect variable E are random variables measurable with respect to the same probability space. Moreover, we assume that both random variables are situated within a **Bayesian network**. That is, there is an acyclic set of directed edges connecting the random variables, representing causal relationships between them. Importantly, this graphical structure satisfies the **Markov condition** with respect to the probability distribution over the variables in the graph: all variables are independent of their non-descendants, conditional on their parents. This setting allows us to calculate the probability distribution over E given each possible intervention setting a set of variables $\boldsymbol{C}$ to each of its possible sets of values, in the style of Pearl (2000). We denote the probability that E takes a value e, given that $\boldsymbol{C}$ is set to a vector of values $\boldsymbol{c}$ via intervention, using the notation $p(e \mid do(\boldsymbol{c}))$. We define the **causal mutual information** between $\boldsymbol{C}$ and E as follows:

$$CMI(\widehat{C}, E) = \sum\nolimits_{c,e} q(c)\, p(e \mid do(c))\, log_2 \frac{p(e \mid do(c))}{p(e)}$$

where q is a probability distribution over possible interventions on $\boldsymbol{C}$.[6]

We aim to define a lexical notion of compression that allows us to meaningfully state that one set of variables $\widehat{\boldsymbol{C}}$ is **more compressed** than another set $\boldsymbol{C}$. Intuitively, we want the relation between more and less compressed variable sets to imply that any distinction captured in the more

[6] The idea of a probability distribution over possible interventions is non-standard, though it does have some precedent in the causal inference literature (Pearl, 1994). In particular, it is necessary to define the information capacity of a causal channel in a way that incorporates an interventional understanding of causation (Ay and Polani, 2008).

compressed variable set $\widehat{\boldsymbol{C}}$ is also captured in the less compressed variable set $\boldsymbol{C}$, but that $\boldsymbol{C}$ can contain distinctions that are not captured in $\widehat{\boldsymbol{C}}$**.** To this end, we begin by noting that for any set of causal variables $\boldsymbol{C}$, we can define a surjective but not injective **compression function** σ from the range of $\boldsymbol{C}$ into another set. This yields a set of variables $\widehat{\boldsymbol{C}}$ whose set of possible values is the range of the function σ. Going forward, we will say that such a variable set $\widehat{\boldsymbol{C}}$ is a **compression** of $\boldsymbol{C}$. We stipulate that for any vector of values $\hat{\boldsymbol{c}}$, the probability $q(\hat{\boldsymbol{c}})$ is given by the equation $\sum_{c\in\sigma^{-1}(\hat{c})} q(\boldsymbol{c})$. Importantly, any probability distribution q over interventions and any other probability distribution p must satisfy the constraint that $p(e\,|\,do(\hat{\mathrm{c}})) = \sum_{c\in\sigma^{-1}(\hat{c})} p(e\,|\,do(c))\,\frac{q(c)}{q(\hat{c})}$.

In other words, the probability that E=e given a coarse-grained intervention setting $\widehat{\boldsymbol{C}}$ to $\hat{\boldsymbol{c}}$ is given by the average, according to q, of each interventional conditional probability $p(e\,|\,do(\boldsymbol{c}))$ for each $\boldsymbol{c}\in\sigma^{-1}(\hat{\boldsymbol{c}})$. Having made these stipulations, we arrive at a lexical, comparative measure of compression: one set of variables $\widehat{\boldsymbol{C}}$ is more compressed than another set $\boldsymbol{C}$ if there is a surjective but not injective compression function σ from the range of $\boldsymbol{C}$ to the range of $\widehat{\boldsymbol{C}}$.[7] In Appendix A, we define the compression relationship between variables more rigorously, but the definition here is sufficient for expositional purposes.

[7] On this lexical measure, it only makes sense to say that one set of variables is more/less compressed than another if the first is a compression of the second or vice versa. We deliberately do not provide a more general measure of compression as such a measure will invariably vitiate on questions not directly relevant to our interests here. For instance, a measure of how compressed the variable set $\boldsymbol{C}$ is that uses the entropy of $\boldsymbol{C}$ will be sensitive to the prior distribution over $\boldsymbol{C}$, but it is unclear why the likelihood that different values of $\boldsymbol{C}$ occur should be relevant to how compressed $\boldsymbol{C}$ is. On the other hand, if we measure how compressed $\boldsymbol{C}$ is by simply counting the number of variable combinations, we lose the ability to say (for example) that a countably infinite discretization of the unit interval results in a more compressed representation. Ultimately, we are interested here in agents' preferences between nested representations of the same causal processes, and so we leave to one side a more general measurement of compression. We believe that comparisons between nested representations deserve special attention, since our choices between different nested representations ultimately determine whether or not any two events observed in our environment should be treated as instantiations of the same causal kind.

We are now in a position to define the amount of information about an effect variable that is lost in the move from a less compressed to a more compressed set of causal variables. We define this quantity as follows:

$$\mathscr{L}(\boldsymbol{C}, \widehat{\mathbf{C}}, E) = CMI(\boldsymbol{C}, E) - CMI(\widehat{\mathbf{C}}, E)$$

Thus, the amount of information about E lost in the move from the less compressed causal representation $\boldsymbol{C}$ to the more compressed representation $\widehat{\boldsymbol{C}}$ is equal to the difference between the causal mutual information that $\boldsymbol{C}$ provides about E and the causal mutual information that $\widehat{\boldsymbol{C}}$ provides about E. In what follows, we will use this equation to derive qualitative predictions about when people will prefer more or less compressed causal representations of their environment, which we then confirm experimentally.

We focus on qualitative comparisons between participants and our model because our goal is to understand the conditions under which people prefer more compressed causal representations of their environment, and our measure provides a principled basis for tractably deriving predictions. Our aim is *not* to directly model the mechanisms by which people estimate compression, and so we leave open the possibility that other measures of compression can be used to derive similar predictions, and may offer more plausible accounts of human cognition at an algorithmic level. Having said this, in what follows we will note some respects in which salient rivals to our framework are less able to explain our data.

Measuring Proportionality Using Information Loss

Having formalized information loss due to compression, we can apply our framework to the compression achieved by replacing a causal variable C with a more compressed variable $\widehat{\boldsymbol{C}}$ and leaving all other variables unchanged. By comparing the amount of information that C communicates about some effect variable E to the amount of information that $\widehat{\boldsymbol{C}}$ communicates about the same variable *E,* we can measure the amount of information that is lost in this compression. This corresponds to a comparison of the causal claims '*C* causes *E*' and '$\widehat{C}$ causes *E*' with respect to their proportionality.

More precisely, let $\mathscr{C} = \left(C_1, \ldots, C_n \right)$ be a sequence of causal variables, with each C_i a compression of all variables $C_{j<i}$. We then say that, in the context of such a sequence, a variable C_i is **proportional** with respect to an effect variable *E* to the extent that $\mathscr{L}\left(\{C_j\}, \{C_i\}, E \right)$ is relatively small for all $j < i$. That is, proportional choices of causal variables are those that preserve information about the conditions under which an effect variable *E* will change, as compared to less compressed alternatives. Note that in this paper we only consider comparisons of proportionality between causal claims with different causal variables and a common effect variable, though one can in principle compare causal relationships that differ with respect to both cause and effect in terms of proportionality. We expect that our use of information loss to measure proportionality generalizes to such comparisons.[8]

Measuring Stability Using Information Loss

Our formalization of information loss due to compression can also be applied to compressions achieved through the omission of background variables. Recall from our earlier discussion that we can measure the stability of a causal relationship $C \rightarrow E$ embedded in a particular causal Bayes net by removing a set of variables $\boldsymbol{B}$ from that Bayes net and assessing how much information is lost in the move from the original Bayes net to the Bayes net that is created by removing the background variables. We stipulate that the variables in a set ***B*** are **background variables** with respect to a causal relationship $C \rightarrow E$ if and only if removing all variables in ***B*** and

[8] Specifically, one could measure the difference between the difference $CME(\ C, E) - CME(\ \widehat{C}, \widehat{E})$, where $\widehat{E}$ is a compression of *E*.

all edges going into or out of the variables in ***B*** creates a Bayesian network that still satisfies the Markov condition. The notion of causal stability can now be made precise, using our proposed measure of information loss. Specifically, we will say that the causal relationship between *C* and *E* is **stable** with respect to background condition ***B*** to the extent that the value of $\mathscr{L}(C, \{\mathrm{C}, \mathbf{B}\}, E)$ is low. That is, the relationship $C \rightarrow E$ is stable with respect to ***B*** to the extent that the average amount of information about *E* that is communicated by interventions on both *C* and the variables in ***B*** is similar to the average amount of information about *E* that is communicated solely by interventions on *C*.

Relationship to Existing Approaches

Our framework is not the first to use information theory to quantify properties of causal relationships. Previous work in this vein includes specific attempts to measure proportionality and stability (Pocheville et al., 2017), as well as attempts to measure other properties of causal relationships, such as power, abstraction, strength, or specificity using formalism from information theory (Ay and Polani, 2008; Beckers and Halpern, 2019; Bourrat, 2021a, 2021b; P. E. Griffiths et al., 2015; Hoel, 2017; Korb et al., 2011). Moreover, our account of information loss can be understood as a version of *rate distortion theory* (Sims, 2016; Zaslavsky et al., 2018), which has been applied to discrete categorization of stimuli and working memory, though not (as far as we know) to causal representation. On this interpretation, the compression function defines a distortion channel, or "information bottleneck" (Tishby et al., 2000), through which information is passed from the less compressed variable set ***C*** to the effect variable *E*, and the function $\mathscr{L}$ measures how much information is lost in the distorting process of compression. Despite these relationships to prior work, none of these approaches argue, as we do, that measurements of the proportionality and stability of a causal relationship can both be expressed in terms of information loss.

Motivation for Experiments 1-2

We have now introduced our theoretical framework for quantifying the amount of information lost in moving from a specific causal representation of a given data-generating process to a more compressed causal representation of that same process. We intend for this framework to serve both a normative and a descriptive role. On the normative side, we have argued above that two putatively positive features of causal claims, proportionality and stability, can be given a unifying account in terms of information loss in causal models. On the descriptive side, we hold that agents, all things being equal, trade off a preference for more compressed causal representations against a preference for causal representations that minimize relative information loss (this descriptive claim summarizes the hypotheses **H1** and **H2** above). These hypotheses can be tested empirically, and we report the results of two experiments that do so in the following two sections.

In order to investigate people's causal representations, we need a measurable response that reflects which variables they are representing and at what level of granularity. Asking people to generate or evaluate Bayes' nets would be a natural approach given our formalism, but doing so would likely require training (see, for example, Bramley et al., 2015, 2017) and this approach would fail to reflect the way that people tend to explicitly represent causal relationships in everyday life. Instead, causal relationships are often expressed in the form of generic causal claims of the form "***C*** causes *E*." Not coincidentally, we have made several such claims in the course of this paper (for example, "smoking causes lung cancer"). Such claims can be taken as evidence for causal representations on the assumption that they bear some systematic relationship to the variables an individual has represented in their internal causal model. Specifically, we take the generation or positive evaluation of a generic causal claim as evidence that the individual producing or endorsing that generic holds a causal model consistent with that claim. For example, suppose that an individual shown data about a fictional insect called the Bricofly evaluates the claim 'raising Bricofly larvae in a warm, humid tank causes them to develop blue wings' more positively than the claim 'raising Bricofly larvae in a warm tank causes them to develop blue wings.' We would take this to be evidence that the individual is representing the environment using a causal model of Bricofly development in which both different temperatures and different humidity levels of the tank are retained in their representation, as opposed to a model in which only the temperature of the tank is represented.

The assumption that the endorsement of generic causal claims reflects an individual's causal representation, which guides our methods in Experiments 1-4, is in keeping with a core tenet of the philosophy of science literature on causation: namely, that generic causal claims are derived from causal models. For instance, Papineau describes himself as "committed to reading 'C causes E' as saying that C is an ancestor of E in a system of generic casual equations," where one can understand a 'system of generic causal equations' as a variety of graphical causal model (Papineau 2022, p. 20). Similarly, Woodward writes that "[causal] modeling techniques are at least sometimes successful in reliably establishing generic causal claims" (Woodward 2019, p. 765). Thus, the current paper's assumption of a close connection between the causal models used to represent a system and generic causal claims about a system is broadly in keeping with at least one branch of the philosophical mainstream.

Experiment 1

In Experiment 1, we test **H1**: that in general, people treat compression as a positive feature of a causal representation, such that all else being equal, the more compressed a given representation is, the better. To test this hypothesis, we presented participants with a description of the results of controlled experiments on a fictional variety of mushroom, fly, or rock, and asked them to rate how good it would be to include various claims in a summary of the described results. These claims included more and less compressed causal claims (e.g., the more compressed claim 'raising Bricofly [a fictional type of insect] larvae in a warm tank causes them to develop blue wings,' and the less compressed claim 'raising Bricofly larvae in a warm, humid tank causes them to develop blue wings'). We manipulated both the vignette used and whether the compression was achieved by coarse-graining a variable (thus manipulating proportionality) or removing a background variable (thus manipulating stability). We predicted that in the absence of information loss due to compression, participants would favor the more compressed representation, and that this would hold for manipulations of both proportionality and stability.

Experiment 1 was also designed to test **H2**: that compression can come at the cost of informativeness, such that the optimal causal representation will achieve a balance of compression and informativeness. To test this, we also manipulated the amount of information loss realized by the more compressed causal claim. For example, if raising Bricofly larvae in a warm, humid tank results in an 85% probability of their developing blue wings, and raising Bricofly larvae in a warm, dry tank results in a 70% probability of their developing blue wings, then implicitly, assuming an equal probability of having a humid or dry tank, the probability of developing blue wings when Bricofly larvae are raised in a warm tank is 77.5%. Under these conditions, and assuming a 1% probability of developing blue wings for larvae raised in a either a cold, humid tank or a cold, dry tank, moving from a causal model that keeps track of the humidity or dryness of the tank to one that does not keep track of these properties results in a loss of information of .01, on the measure of information loss that we introduce above.[9] Thus, by manipulating the conditional probabilities in the data sets shown to participants, we were able to manipulate the amount of information loss inherent in endorsing a more compressed causal claim. We predicted that with greater information loss, we would see lower evaluations of the more compressed representation relative to the less compressed representation.

The data, stimuli, and pre-registrations for all experiments in this paper are available at https://osf.io/zm6kr/?view_only=124c22b8b2dd4d64b44046c8784911db.

Participants

Participants were 450 adults recruited via Prolific. An additional 150 participants were excluded for failing comprehension checks or for rating poor causal claims non-negatively. The sample of participants was 49.6% female and 48.9% male, with an age range of 19-79 and a mean age of 40.[10] For all studies reported here, participation was restricted to users with a US-based IP address and a 95% rating based on at least 100 previous studies. All studies received IRB approval from the authors' University.

[9] Here, we give explicit calculations of information loss for x=.7 and x=.85 to illustrate how these values are calculated. When x=.7, the probability p(Blue Wings) = $.25[.7+.7+.01+.01]=.355$. The causal mutual information between the less compressed causal variable and the binary variable for whether a Bricofly develops blue wings is $.25\left[.7\log_2\frac{.7}{.355}+.7\log_2\frac{.7}{.355}+.01\log_2\frac{.01}{.355}+.01\log_2\frac{.01}{.355}+.3\log_2\frac{.3}{.645}+.3\log_2\frac{.3}{.645}+.99\log_2\frac{.99}{.645}+.99\log_2\frac{.99}{.645}\right]\approx .47$. The causal mutual information between the more compressed causal variable and the binary Blue Wings variable is $.5\left[.7\log_2\frac{.7}{.355}+.01\log_2\frac{.01}{.355}+.3\log_2\frac{.3}{.645}+.99\log_2\frac{.99}{.645}\right]\approx .47$ so that the information lost due to compression is zero. When x=.85, p(Blue Wings) = $.25[.85+.7+.01+.01]=.3925$ and so the causal mutual information between the less compressed causal variable and the binary variable for whether a Bricofly develops blue wings is $.25\left[.85\log_2\frac{.85}{.3925}+.7\log_2\frac{.7}{.3925}+.01\log_2\frac{.01}{.3925}+.01\log_2\frac{.01}{.3925}+.15\log_2\frac{.15}{.6075}+.3\log_2\frac{.3}{.6075}+.99\log_2\frac{.99}{.6075}+.99\log_2\frac{.99}{.6075}\right]\approx .55$, while the causal mutual information between the more compressed causal variable and the binary Blue Wings variable is $.5\left[.775\log_2\frac{.775}{.3925}++.01\log_2\frac{.01}{.3925}+.225\log_2\frac{.225}{.6075}+.99\log_2\frac{.99}{.6075}\right]\approx .54$, yielding an information loss due to compression of approximately .01.

[10] Demographic data obtained from participants was not solicited as part of our study, but was provided by participants to Prolific upon joining the platform.

Materials and Procedures

Participants read a vignette in which they learned about a novel causal system, including the results of experiments involving that system. For example, in the insect vignette, participants were presented with one of the following reports of results of experiments on the fictional "Bricofly":

Report 1:

a) x% of all Bricofly larvae raised in a warm, humid tank developed blue wings;

b) 70% of all Bricofly larvae raised in a warm, dry tank developed blue wings;

c) 1% of all Bricofly larvae raised in a cold, humid tank developed blue wings;

d) 1% of all Bricofly larvae raised in a cold, dry tank developed blue wings.

Report 2:

a) x% of all Bricofly larvae raised in a warm tank and sprayed with water developed blue wings;

b) 70% of all Bricofly larvae raised in a warm tank and blown with dry air developed blue wings;

c) 1% of all Bricofly larvae raised in a cold tank and sprayed with water developed blue wings;

d) 1% of all Bricofly larvae raised in a cold tank and blown with dry air developed blue wings.

The value of *x* was varied between subjects and set at either 70, 85, or 98. These values correspond to information loss amounts of 0, .01, and .06 respectively in moving from a less compressed representation to a more compressed representation, assuming a uniform distribution over possible interventions on the causal variable(s) in the less compressed models in which the evaluated causal claims can be represented.

Table 1

Structure of Vignettes used in Experiments 1 and 2.

Vignette	Effect	Primary Cause	Secondary Cause	Background Condition
Drol (Mushroom)	Bumpy Stems	High/Low Mineral Soil	High/Low Sodium Soil	Watered with Salt/Fresh Water
Bricofly (Insect)	Blue Wings	Warm/Cold Tank	Humid/Dry Tank	Water Spray/Dry Air Blow
Chapagite (Rock)	Fissures	Warm/Cold Water	Salt/Fresh Water	Wrapped in Saline/Plain Cloth

After receiving a version of the findings described above, participants were then asked to rate, on a scale from -3 (very bad) to 3 (very good), "how good it would be to include each of the following statements in a summary of this report:"

- *Compressed*: Raising Bricofly larvae in warm tank causes them to develop blue wings.
- *High*: Raising Bricofly larvae [in a warm, humid tank/in a warm tank and spraying them with water] causes them to develop blue wings.
- *Low*: Raising Bricofly larvae [in a warm, dry tank/in a warm tank and blowing them with air] causes them to develop blue wings.

The causal claims *High* and *Low* are so-named because the cause cited in *High* always confers the same or greater probability onto the effect than the causal claim *Low* (e.g., it is always the case that $p(\text{Blue Wings} \mid \text{Warm, Humid Tank}) \geq p(\text{Blue Wings} \mid \text{Warm, Dry Tank})$).

As an attention check, participants were also asked to rate the following causal claims:

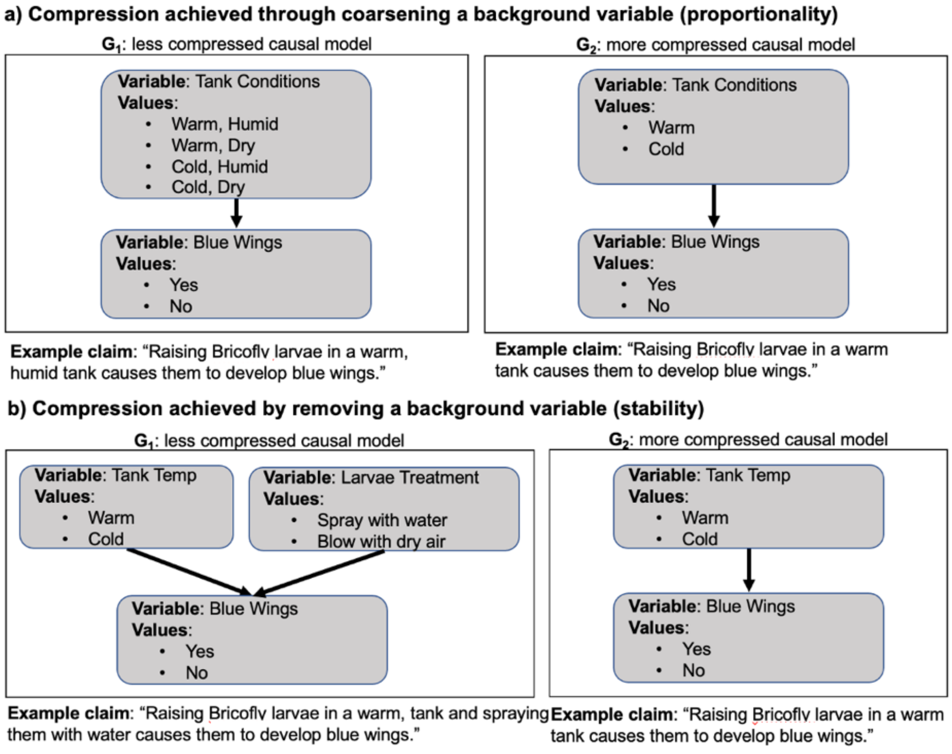
- *Compressed (Bad):* Raising Bricofly larvae in a cold tank causes them to develop blue wings.

- *High (Bad)*: Raising Bricofly larvae [in a cold, humid tank/in a cold tank and spraying them with water] causes them to develop blue wings.
- *Low (Bad)*: Raising Bricofly larvae [in a cold, dry tank/in a cold tank and blowing them with air] causes them to develop blue wings.

Since all of these claims cite a cause that *lowers* the probability of its effect, we take them to be infelicitous in the context of the scenario shown to participants. Thus, participants who gave ratings at or higher than the scale midpoint (i.e., a rating of 0-3) were excluded.

Figure 1

Graphs Showing the Causal Relationships between Variables in Experiment 1


a) Compression achieved through coarsening a background variable (proportionality)
G1: less compressed causal model
Variable: Tank Conditions
Values:
Warm, Humid
Warm, Dry
Cold, Humid
Cold, Dry
Variable: Blue Wings
Values:
Yes
No
Example claim: "Raising Bricofly larvae in a warm, humid tank causes them to develop blue wings."
G2: more compressed causal model
Variable: Tank Conditions
Values:
Warm
Cold
Variable: Blue Wings
Values:
Yes
No
Example claim: "Raising Bricofly larvae in a warm tank causes them to develop blue wings."
b) Compression achieved by removing a background variable (stability)
G1: less compressed causal model
Variable: Tank Temp
Values:
Warm
Cold
Variable: Larvae Treatment
Values:
Spray with water
Blow with dry air
Variable: Blue Wings
Values:
Yes
No
Example claim: "Raising Bricofly larvae in a warm, tank and spraying them with water causes them to develop blue wings."
G2: more compressed causal model
Variable: Tank Temp
Values:
Warm
Cold
Variable: Blue Wings
Values:
Yes
No
Example claim: "Raising Bricofly larvae in a warm tank causes them to develop blue wings."

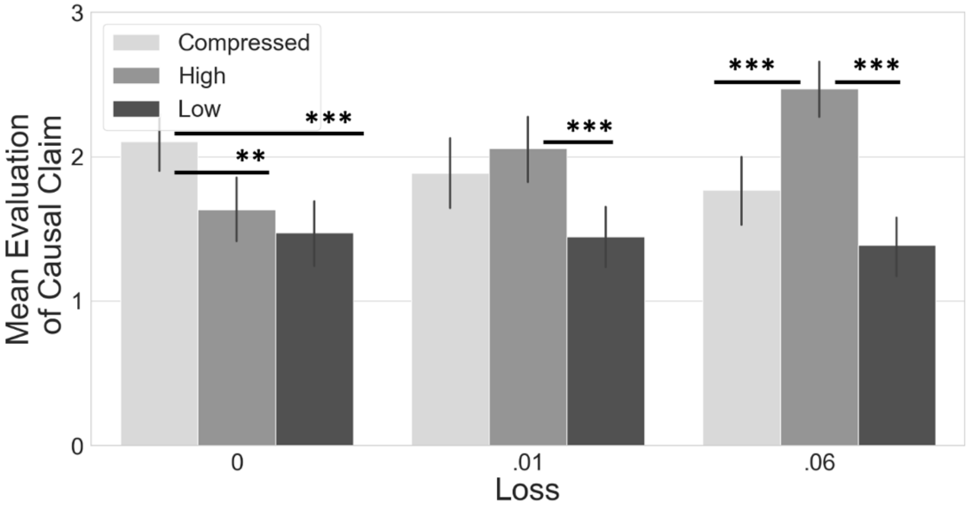
Note. The top panel shows the compression used in the proportionality condition, and the bottom panel shows the compression used in the stability condition. In both panels, the right graph shows the more compressed causal model in which the claim *Compressed* is embedded, and the left graph shows the less compressed causal model in which the claims *High* and *Low* are embedded.

For participants shown Report 1, the claim *Compressed* is a compression achieved by coarsening a causal variable, thus varying proportionality. For participants shown Report 2, the claim *Compressed* is a compression achieved by eliding a background variable, thus varying stability.[11] Figure 1 shows the implicit causal model in which each of these claims is embedded.

Results

Figure 2

Evaluations of Causal Claims as a Function of Information Loss



Note. Bar plots showing mean evaluations in Experiment 1 (with 95% confidence intervals) of causal claims under different loss conditions for all participants. Participants were asked to rate "how good it would be to include each of the following statements in a summary of this report," with ratings ranging from -3 (very bad) to 3 (very good). Double asterisks indicate significant within-participants differences at the .01 level in a mixed effects ANOVA, and triple asterisks indicate significant within-participants differences at the .001 level. Specifically, mixed ANOVA for each value of Loss found that at Loss=0, Compressed was rated more highly than both High ($\eta^2 = .025$, $p = .002$) and Low ($\eta^2 = .041$, $p < .001$). When Loss=.01, High was not rated significantly higher than Compressed ($\eta^2 = .005$, $p = .156$), but was rated higher than Low ($\eta^2 = .050$, $p < .001$). When Loss=.06, High was rated significantly higher than both Compressed ($\eta^2 = .058$, $p < .001$) and Low ($\eta^2 = .153$, $p < .001$).

[11] As described in the previous section, on our formal analysis, both the coarsening of a causal variable and the elision of a background condition can both be expressed as compressions of a partition over the same sample space, such that there is no difference between these two kinds of compression. We have fashioned our examples to match what is understood in the literature (e.g., Woodward, 2010) as a distinction between a refinement of the same variable and an elision of a background condition.

Figure 2 presents the mean ratings for each of *Compressed*, *High*, and *Low* as a function of the amount of information loss incurred through compression. To test whether evaluation of less compressed causal claims relative to more compressed causal claims increased as a function of information loss due to compression, we computed (as pre-registered) two difference scores:

- COMPRESSED-HIGH. The difference between the participant's evaluation of *Compressed* and their evaluation of *High* (e.g., the difference between the evaluation of 'Raising Bricofly larvae in a warm tank causes them to develop blue wings' and the evaluation of 'Raising Bricofly larvae in a warm, dry tank causes them to develop blue wings'). This reflects the degree to which an agent sees a causal claim derived from a more compressed model as more fitting than the most compelling causal claim that can be derived from a less compressed model.
- COMPRESSED-AVG(HIGH, LOW). The difference between the participant's evaluation of *Compressed* and a uniform average of their evaluations of *High* and *Low* (e.g., the difference between the evaluation of 'Raising Bricofly larvae in a warm tank causes them to develop blue wings' and the average evaluation of 'Raising Bricofly larvae in a warm, humid tank causes them to develop blue wings' and 'Raising Bricofly larvae in a warm, dry tank causes them to develop blue wings').[12] This reflects the degree to which an agent sees a causal claim derived from a less compressed model as more fitting, on average, than *any* causal claim that can be derived from a less compressed model.

We regressed these dependent variables against independent variables denoting the assigned vignette (Vignette), whether the more compressed claim is achieved by coarsening a causal

[12] Due to an error, the equation for COMPRESSED-AVG(HIGH, LOW) was pre-registered for both experiments as Evaluation of *Compressed* - .5(Evaluation of *High*-Evaluation of *Low*). However, the correct equation is Evaluation of *Compressed* - .5(Evaluation of *High*+Evaluation of *Low*).

variable or removing a background variable (Mode of Compression), and the amount of information loss inherent in moving from the less-compressed to more-compressed causal model in which causal claims are embedded (Loss), as well as all possible interactions between the independent variables. The regressions revealed that only Loss was a significant predictor of COMPRESSED-HIGH ($\beta = -16.623$, $p < .001$,$R^2 = .083$), so that the more information that was lost in the move from a more detailed to a more compressed causal model, the more participants preferred the claim *High* to the claim *Compressed*. Loss was also the only significant predictor of COMPRESSED-AVG(HIGH, LOW) ($\beta = -9.852$, $p < .001$,$R^2 = .041$). Notably, we found no evidence of a significant interaction between Loss and Mode of Compression on these dependent variables (COMPRESSED-HIGH: $\beta = -.426$, $p = .875$, $R^2 = .083$; COMPRESSED-AVG(HIGH, LOW): $\beta = -2.106$, $p = .426$, $R^2 = .041$), nor did we find any significant interaction effects between Mode of Compression and any other independent variables.

For additional analyses based on each individual rating (*Compressed*, *High*, *Low*), see Figure 2 as well as Supplementary Materials. As a sanity check, we also analyzed the difference between participants' evaluation of *High* and their evaluation of *Low* (we label this difference 'HIGH-LOW'). As expected, only Loss was a significant predictor of HIGH-LOW ($\beta = 13.543$, $p < .001$, $R^2 = .130$), with larger values for HIGH-LOW as the probability of the effect given the cause in *High* increased (resulting in greater information loss).

In an exploratory analysis, we calculated the percentage of participants who strictly preferred *Compressed* to *High* across all three loss levels. This percentage was approximately 36% when Loss=0, 21% when Loss=.01, and 10% when Loss=.06.

Discussion

These results provide strong evidence in favor of the claim that participants' relative evaluations of more and less compressed causal claims are partially governed by the amount of information loss that is inherent in the more compressed causal claim. When there is no information loss, participants evaluate more compressed causal claims significantly more highly than less compressed causal claims (consistent with **H1**), suggesting that people award simplicity and penalize unnecessary complexity in their evaluation of causal claims. When information loss is moderate, there is no significant difference between participants' evaluations of more and less compressed causal claims, suggesting that some participants prefer a compressed claim even when some information loss is inherent in compression (consistent with **H2**).

Importantly, we found that participants' pattern of evaluation of causal claims was similar across the condition in which compression was achieved by coarsening a causal variable and the condition in which compression was achieved by removing a background variable (see Supplementary Materials for additional figures by Mode of Compression). In keeping with our analysis, this suggests that the amount of information that is lost due to compression is related to evaluations of the quality of causal claims in the same way across both of these ways of compressing a causal representation of one's environment. This, in turn, is in keeping with our unified analysis of judgements of the proportionality and stability of causal claims in terms of information loss.

Experiment 2

In Experiment 1, participants evaluated the three key causal claims (*Compressed*, *High*, and *Low*) on the same screen. This could have introduced unintended task demands. For instance, participants may have felt that endorsing *Compressed* was redundant with the endorsement of both

High and *Low*, or that endorsing *Compressed* (when the option to select more fine-grained options was available) implied the causal irrelevance of the unspecified factor. To ensure that the results of Experiment 1 were robust to such considerations, we replicated the study with the amendment that participants were shown the same data twice, and first asked to evaluate *Compressed*, and only asked to evaluate *High* and *Low* after their evaluation of *Compressed* was completed.

Participants

Participants were 483 adults recruited via Prolific. An additional 117 participants were excluded for failing comprehension checks or rating poor causal claims non-negatively. The sample of participants was 50.1% female and 48.6% male, with an age range of 19-81 and a mean age of 38.

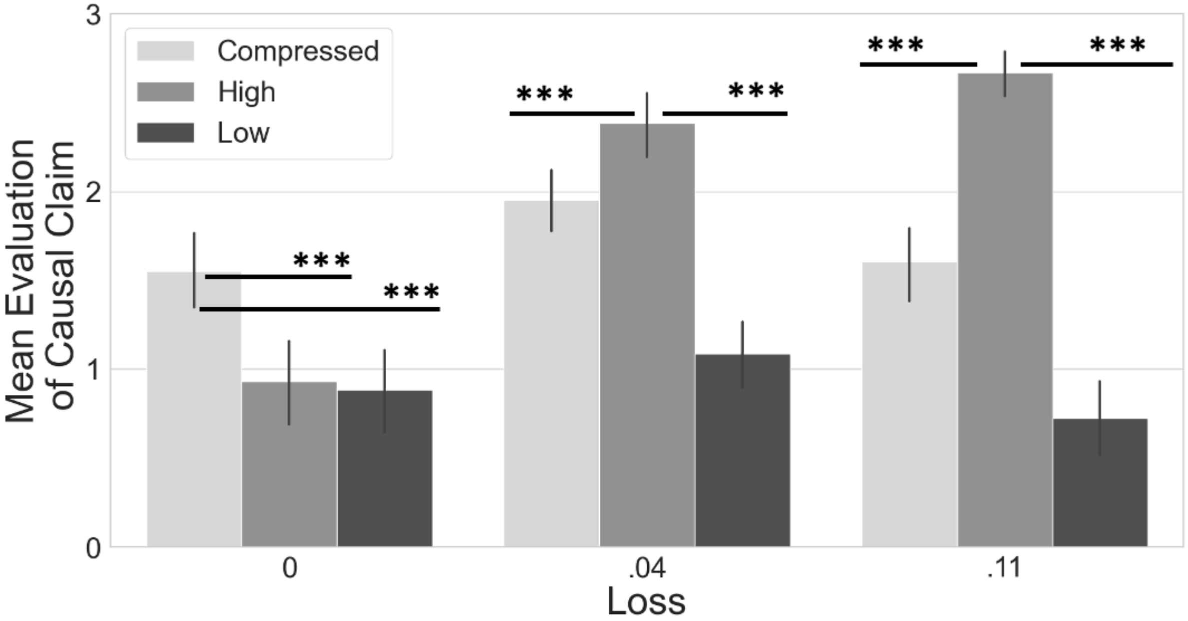
Materials and Procedures

The procedure was identical to that used in Experiment 1 with three exceptions. First, as described above, participants completed their evaluation of *Compressed* before being asked to evaluate *High* and *Low*. Second, sentence (b) in both descriptions used in the first experiment was amended to replace 70% with 55%. Analogous replacements were made for the other two vignettes. Third, the value of x in sentences (a) and (b) was varied between participants and was set at either 55, 85, or 98, leading to information loss amounts of 0, .04, and .11 respectively, under the assumption of a uniform distribution over possible interventions on causes in the less-compressed representation of the data-generating process. Thus, Experiment 2 replicates Experiment 1 for a different range of loss values.

Results

Figure 3

Evaluations of Causal Claims as a Function of Information Loss



Note. Bar plots showing mean evaluations in Experiment 2 (with 95% confidence intervals) of causal claims under different loss conditions for all participants. Triple asterisks indicate significant within-participants differences at the .001 level. Mixed ANOVA for each value of Loss found that at Loss=0, Compressed was rated more highly than both High ($\eta^2 = .044$, $p < .001$) and Low ($\eta^2 = .048$, $p < .001$). At Loss=.04, High was rated more highly than Compressed ($\eta^2 = .036$, $p < .001$) and Low ($\eta^2 = .220$, $p < .001$). At Loss=.11, High was rated more highly than Compressed ($\eta^2 = .179$, $p < .001$) and Low ($\eta^2 = .442$, $p < .001$).

Figure 3 presents the mean ratings for each of *Compressed*, *High*, and *Low* as a function of Loss. We performed the same regressions as in Experiment 1. Loss was a significant predictor of all three dependent variables (COMPRESSED-HIGH: $\beta = -14.543$, $p < .001, R^2 = .213$; COMPRESSED-AVG(HIGH, LOW): $\beta = -6.391$, $p < .001$, $R^2 = .064$; HIGH-LOW: $\beta = 16.303$, $p < .001$, $R^2 = .296$). Thus, as the amount of information lost due to compression increased, *Compressed* was again evaluated more negatively in comparison with more detailed causal claims. In addition, we again saw larger values for HIGH-LOW as the probability of the effect given the cause in *High* increased (resulting in greater information loss). For additional analyses based on each individual rating (*Compressed*, *High*, *Low*), see Figure 3 and Supplementary Materials.

In a further replication of Experiment 1, the manner in which compressed causal claims were generated (i.e., either by coarsening a causal variable or removing a background variable) was not a significant predictor of any of the three dependent variables measured, nor did it interact with Loss (Regression statistics for the interaction between Mode of Compression and Loss: COMPRESSED-HIGH: $\beta = -1.803, p = .171, R^2 = .213$; COMPRESSED-AVG(HIGH, LOW): $\beta = -1.172$, $p = .327, R^2 = .064$; HIGH-LOW: $\beta = 1.263$, $p = .281$, $R^2 = .296$; see Supplementary Materials for additional figures broken down by Mode of Compression). This again suggests that measures of information loss in causal model compression provide a unifying account of the value of both proportional and stable causal claims.

In an exploratory analysis, we measured the percentage of participants who strictly preferred *Compressed* to *High* across all three loss levels. This percentage was approximately 39% when Loss=0, 10% when Loss=.04, and 2% when Loss=.11.

The results of Experiment 2 replicate the positive results of Experiment 1 for a different range of loss levels, and under conditions such that *Compressed* was evaluated separately from *High* and *Low*. This renders the aforementioned concerns about task demands less plausible.

The results of Experiment 2 also offer evidence against an alternative interpretation of our results. Specifically, the *causal power* theory (Cheng, 1997) holds that agents evaluate causal claims positively to the extent that they optimize the following quantity, which we express in terms of Pearl's do-calculus:

$$\text{Power}(c, e) = \frac{p(e|do(c)) - p(e|do(\neg c))}{1 - p(e\,|\,do(\neg c))}.$$[13]

Although the theory is not typically applied to differences between causal claims, but rather to the distinction between causal and non-causal associations, a natural application to our current case is to consider the difference in causal power between less compressed and more compressed causal claims as a measure of the extent to which the less compressed claim should be preferred. When we do so, however, the causal power theory predicts different results for the dependent variable COMPRESSED-AVG(HIGH, LOW) than those observed in Experiment 2. To illustrate, see Table 2, which shows the power of each of the three causal claims evaluated in Experiment 2, and the resulting value of Power(Comp) - AVG[Power(High), Power(Low)].

Table 2

Causal power values for causal claims in Experiment 2.[14]

p(Effect|High)	Power(Comp)	Power(High)	Power(Low)	Power(Comp) - AVG[Power(High),Power (Low)][15]
.55	.545	.444	.444	.101
.85	.697	.815	.366	.106
.98	.763	.975	.325	.112

If evaluations of causal claims are primarily driven by differences in causal power, then we would expect that the difference between participants' evaluations of *Compressed* and their average evaluation of *High* and *Low* (i.e., the dependent variable COMPRESSED-AVG(HIGH, LOW)) should be positively correlated with value of Power(Comp) - AVG[Power(High),

[13] The original formulation of causal power given in Cheng (1997) is not stated in terms of Pearl's do-calculus. Instead, it is written $\text{Power}(c, e) = \frac{p(e|c) - p(e|\neg c)}{1 - p(e\,|\,\neg c)}$ (see p. 374, Eq. 8). We state causal contrast in these terms here to maintain formal consistency with our own measure of information loss.

[14] To demonstrate how causal power values were calculated, when p(Effect|High)=.85, Power(Comp) = (.5(.85+.55)-.01)/(1-.01) = .697, whereas Power(High) = (.85 – (1/3)(.55 + .01 + .01))/(1-(1/3)(.55 + .01 + .01)) = .815.

[15] The figures in this column are calculated using unrounded values of the values listed in other columns.

Power(Low)]. However, if we use the data from Experiment 2 to regress COMPRESSED-AVG(HIGH, LOW) against Power(Comp) - AVG[Power(High), Power(Low)], along with Mode of Compression, Vignette, and all interactions between these three variables, we observe a significant predictive relationship between COMPRESSED-AVG(HIGH, LOW) and Power(Comp) - AVG[Power(High), Power(Low)] going in the *opposite* direction ($\beta = -65.853$, $p = .001$, $R^2 = .066$), such that higher values of Power(Comp) - AVG[Power(High), Power(Low)] are associated with *lower* values of COMPRESSED-AVG(HIGH, LOW). Thus, a causal power theory fails to predict a crucial dependent variable that our information loss theory is able to successfully predict.

Discussion

Nevertheless, our results in Experiments 1 and 2 remain subject to two salient concerns. First, Experiments 1-2 varied proportionality by adding or omitting qualifiers to a variable (e.g., warm *humid* tank versus warm tank). More canonical manipulations of proportionality involve a *continuum* that can be coarsened into discrete ranges (e.g., a scale with ten values that is coarsened into two ranges of values). Thus, the results of Experiments 1-2 leave open the possibility that these more canonical manipulations of proportionality would show divergence between effects of information loss on proportionality and stability. Second, Experiments 1 and 2 were designed to provide positive support for H1 and H2, but were not designed to differentiate our account from another alternative hypothesis: that evaluations of more and less compressed causal claims do not reflect information loss, as our account suggests, but instead differences in *causal contrast* (consistent with Lien and Cheng, 2000). Experiment 3 was designed to addresses both of these concerns.

Experiment 3

Lien and Cheng (2000) develop an account of how people differentiate between genuine and spurious causes, and in so doing present results in keeping with the claim that when agents choose between candidate causal explanations of a given event, they choose the one that maximizes *causal contrast*, which is given by the following equation:[16]

$$\text{Cont}(e; c) = p(e \mid do(c)) - p(e \mid \text{do}(\neg c)).$$

Applying this formula to the values from Experiments 1 and 2 reveals that as the probability of the effect for *High* increases, the difference in contrast between the compressed causal claim and the high causal claim decreases.[17] So, it seems that our results in Experiment 1-2 might just as well be explained by the hypothesis that participants are basing their judgments on the difference in contrast between *Compressed* and *High* as they are by our hypothesis that participants are balancing compression against information loss. To distinguish between these two hypotheses, we ran an experiment using a similar paradigm to Experiments 1-2, but wherein participants were shown data sets for which information loss and causal contrast generated different qualitative predictions. This design allows us to test which of these two quantities is a more plausible candidate for the cue that participants are using to evaluate causal claims. Experiment 3 also

[16] As in the case of causal power, we have re-written Lien and Cheng's contrast measure in terms of Pearl's do-calculus. In the original formulation, causal contrast is written as $\text{Cont}(e; c) = p(e \mid c) - p(e \mid \neg c)$.

[17] In Experiment 1, when *x=.7*, one can calculate the key contrast figures as follows: $\text{Cont}(\text{Blue Wings; Warm Tank}) = .7 - .01 = .69$, $\text{Cont}(\text{Blue Wings; Warm, Humid Tank}) = .7 - \frac{1}{3}[.7 + .01 + .01] = .46$, $\text{Cont}(\text{Blue Wings; Warm Tank}) - \text{Cont}(\text{Blue Wings; Warm, Humid Tank}) = .23$. When *x=.85*, the key contrast calculations are: $\text{Cont}(\text{Blue Wings; Warm Tank}) = \frac{1}{2}[.85 + .7] - .01 = .765$, $\text{Cont}(\text{Blue Wings; Warm, Humid Tank}) = .85 - \frac{1}{3}[.7 + .01 + .01] = .61$ $\text{Cont}(\text{Blue Wings; Warm Tank}) - \text{Cont}(\text{Blue Wings; Warm, Humid Tank}) = .155$. When *x=.98*, the key contrast calculations are: $\text{Cont}(\text{Blue Wings; Warm Tank}) = \frac{1}{2}[.98 + .7] - .01 = .83$, $\text{Cont}(\text{Blue Wings; Warm, Humid Tank}) = .98 - \frac{1}{3}[.7 + .01 + .01] = .74$, and $\text{Cont}(\text{Blue Wings; Warm Tank}) - \text{Cont}(\text{Blue Wings; Warm, Humid Tank}) = .09$.

differed from Experiments 1-2 in manipulating proportionality through coarsenings of a continuous quantity.

Participants

Participants were 458 adults recruited via Prolific. An additional 185 participants were excluded for failing comprehension checks or rating poor causal claims non-negatively. The sample of participants was 49.9% female and 50.1% male, with an age range of 18-79 and a mean age of 37.

Materials and Procedures

Participants read a vignette in which they learned about a novel causal system, including the results of experiments involving that system. As in Experiments 1 and 2, the fictional experiments involved either insects, mushrooms, or rocks. For example, in the insect vignette, participants assigned to the proportionality condition were presented with one of the data scenarios shown in Table 3, and asked to evaluate the following three causal claims on a scale from -3 to 3:

- *Compressed*: Raising Bricofly larvae in a moderate-temperature tank causes them to develop blue wings.
- *High*: Raising Bricofly larvae in a moderately warm tank causes them to develop blue wings.
- *Low:* Raising Bricofly larvae in a moderately cold tank causes them to develop blue wings.

The causal claims evaluated in the stability case were identical to those evaluated in Experiments 1 and 2, only with “moderate-temperature tank” replacing “warm tank” in the insect vignette, and a similar substitution made in other vignettes. Table 3 also shows the values of both information loss and the difference in contrast between *Compressed* and *High* for all three data sets. As can be seen from the table, Scenarios 2 and 3 both differ from Scenario 1 by the same amount with respect to the difference in contrast between *Compressed* and *High*, but only Scenario 2 differs from Scenario 1 with respect to information loss. Thus, if we believe that information loss and not causal contrast is affecting participants' evaluations of causal claims, then we would predict that participants will treat Scenarios 1 and 3 similarly, but treat Scenario 2 differently from both Scenarios 1 and 3.

Table 3

Information loss and causal contrast for three different scenarios shown in Experiment 3.

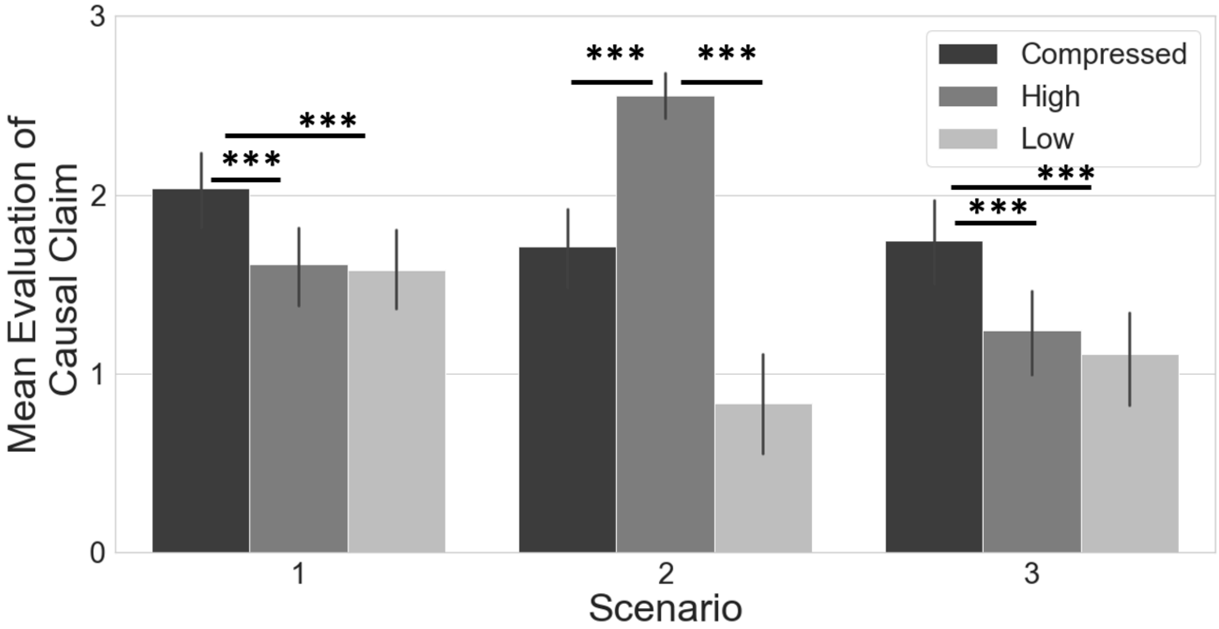
Scenario 1		***Scenario 2***		***Scenario 3***	
Tank Condition	**% of Bricofly Developing Blue Wings**	**Tank Condition**	**% of Bricofly Developing Blue Wings**	**Tank Condition**	**% of Bricofly Developing Blue Wings**
Extremely Cold Tank (0-24 degrees)	1%	Extremely Cold Tank (0-24 degrees)	1%	Extremely Cold Tank (0-24 degrees)	43%
Moderately Cold Tank (25-49 degrees)	70%	Moderately Cold Tank (25-49 degrees)	70%	Moderately Cold Tank (25-49 degrees)	70%
Moderately Warm Tank (50-74 degrees)	70%	Moderately Warm Tank (50-74 degrees)	98%	Moderately Warm Tank (50-74 degrees)	70%
Extremely Warm Tank (55-99 degrees)	1%	Extremely Warm Tank (55-99 degrees)	1%	Extremely Warm Tank (55-99 degrees)	43%
Loss	0	**Loss**	.06[18]	**Loss**	0
Contr(Comp)-Contr(High)	.23	**Contr(Comp)-Contr(High)**	.09	**Contr(Comp)-Contr(High)**	.09

[18] In the pre-registration for Experiment 3, this figure is mistakenly given as '.31.'

Results

Figure 4

Mean evaluations of causal claims for three scenarios in Experiment 3.



Note. Triple asterisks indicate significant within-participants differences at the .001 level.

Figure 4 shows the results of Experiment 3 for all three scenarios across both modes of compression. As we were primarily concerned with differential evaluations of *Compressed* and *High* across different scenarios, we ran mixed ANOVA for the within-participants difference between *Compressed* and *High* in each scenario. We found that in Scenarios 1 and 3, *Compressed* was strictly preferred to *High* (Scenario 1: $\eta^2 = .019, p = .001$; Scenario 3: $\eta^2 = .032, p < .001$). This is consistent with **H1** (since participants favored compression when information loss was zero). Moreover, the preference for the claim *Compressed* over the claim *High* did not differ across Scenarios 1 and 3 ($\beta = .041$, $p = .637$), which is consistent with the predictions of information loss, but not those of causal contrast (see Table 3).

Unlike Scenarios 1 and 3, in Scenario 2, the causal claim *High* was strictly preferred to the causal claim *Compressed* ($\eta^2 = .120, p < .001$). This is consistent with the hypothesis that compression trades off with information loss, such that less compressed causal claims may be favored when information loss is not negligible. Moreover, the difference in ratings between *Compressed* and *High* differed across Scenarios 2 and 3 ($\beta = -22.276$, $p < .001$, $R^2 = .232$); this is consistent with the predictions of information loss, but not with those of causal contrast (see Table 3).

Notably, we do see a significant effect of the proportionality/stability condition on the difference in evaluations between *Compressed* and *High* in Scenario 1 ($\eta^2 = .031, p = .006$), but not for Scenarios 2 and 3, suggesting that the finding for Scenario 1 is likely spurious. A full report of all pre-registered analyses for Experiment 3 is provided in the Supplemental Materials.

Discussion

Although Lien and Cheng (2000)'s causal contrast theory was developed as an account of how people differentiate between genuine and spurious causes, rather than how people determine a level of compression at which to represent the causal structure of their environment, causal contrast nevertheless offers a natural alternative to our own account of information loss in explaining why people might favor more or less compressed causal claims. Experiment 3 was designed to provide a direct test of the predictions of information loss versus those of causal contrast in explaining judgments like those elicited in Experiments 1-3. The results provide clear support for information loss: differences in information loss (holding differences in causal contrast fixed) predicted different patterns in ratings, while differences in causal contrast (holding information loss fixed) did not. Moreover, because our proportionality cases in Experiment 3 involve coarsenings of a continuous quantity (e.g., temperature), the results bolster our claim that the information loss framework provides a unified account of evaluations of both proportionality and stability.

In another experiment with a similar design to Experiment 3, we directly tested whether our theory could better predict peoples' evaluations of causal claims than a causal power theory. While the results of that experiment are less conclusive, we argue that on the whole (including the results of Experiment 2), our results are better explained by our information loss-based theory than a causal power-based theory. The materials, results, and analysis of this experiment are reported in the Supplemental Materials.

Having established our first key hypotheses (**H1** & **H2**) – that people treat both informativeness and compression as positive features of causal claims, to be traded off against one another – we turn to our third key hypothesis (**H3**) – that agents will tolerate information loss for the sake of compression whenever the lost information is not decision-theoretically valuable to that agent in a given context.

Decision Theory and The Value of Lost Information

The results of Experiments 1-3 demonstrate that the strength of participants' preference for more compressed causal claims over less compressed claims is predicted by the amount of information loss achieved by moving from a model in which the less compressed claim is embedded to a model in which the more compressed claim is embedded. However, these results also show that when the total amount of information loss is low, the preference for more detailed causal claims over less detailed ones is not significant. Additionally, they show that even as the total amount of information loss increases, mean evaluations of compressed causal claims remain positive. This suggests that, all things considered, people assign value to *lossy compressions* of a causal model, even when they have the opportunity to endorse a lossless compression of the same underlying data, where a "lossless" compression is understood as one that preserves all four variable values.

What explains the value that people assign to compressed causal claims that elide information about the underlying dynamics of the processes that they represent? As stated in the introduction, our hypothesis is that agents' evaluations of causal claims that compress their target systems with information loss are driven at least in part by agents' judgments as to the *decision-theoretic value* of the information that is lost in compression. That is, when agents judge that the information that is lost in the move to a more compressed causal model is *not relevant* to their choice between a set of feasible actions, they evaluate the compressed causal claim more positively than they would if the lost information *were* relevant to their choice between a set of actions.

Table 4

Data shown to participants in Experiment 4.

Medication	Facts about Medication (Proportionality Condition)	Facts about Medication (Stability Condition)	% of Patients with Reduced Severity of Headaches
A	Active ingredient Type-1 Reptol	Active ingredient Reptol and taken with food	x% (x > 80)
B	Active ingredient Type-2 Reptol	Active ingredient Reptol and taken without food	70%
C	Active ingredient Type-1 Psylo	Active ingredient Reptol and taken with food	1%
D	Active ingredient Type-2 Psylo	Active ingredient Psylo and taken without food	1%

To illustrate, consider an agent who is tasked with stocking headache medicines at a pharmacy. These medicines can have one of two ingredients (Reptol or Psylo), each of which comes in one of two types (Type-1 or Type-2). The agent wants to stock all and only those headache medicines that relieve headaches in *y%* of patients, and has access to the data in Table 4. Suppose that the agent is in the proportionality condition. If *y*=50, such that they would stock any medicine that reduces severity of headaches in at least 50% of patients, then whether a Reptol-based or Psylo-based medicine is of Type-1 or Type-2 is irrelevant to their decision; they will stock Reptol-based medicines and not Psylo-based medicines. Thus, the compression from 'Type-1 Reptol causes reduced severity of headaches' to 'Reptol causes reduced severity of headaches' incurs no loss of decision-theoretically valuable information for this agent; it elides only decision-irrelevant information about the type of Reptol, and we would expect the agent to be indifferent between the two claims if asked to evaluate their aptness in describing the efficacy of headache medicines more generally, or to show a preference for the more compressed claim. By contrast, if y=80, such that the agent would stock any medicine that reduces severity of headaches in at least 80% of patients, and if x>80, then only Type-1 Reptol should be stocked, according to the agent's own preferences. Thus, the compression from 'Type-1 Reptol causes reduces severity of headaches' to 'Reptol causes reduces severity of headaches' *does* elide decision-relevant information about the type of Reptol. Under this condition, we would therefore expect an agent to evaluate the less compressed causal claim more positively than the more compressed causal claim. We use this motivating example as part of our materials in Experiment 4.

Mathematically, we can define the value of the information lost when moving from a less compressed causal representation to a more compressed representation as follows. We begin with a set of variables that can be partitioned into a singleton set containing an **action variable** A, a singleton set containing an **effect variable** E, and a subset of **observable variables** $\boldsymbol{O}$. Let $u{:}R_A \times R_E \rightarrow \mathbb{R}$ be a real-valued utility function defined on the range of the action variable A and the effect variable E. Each value $u(a, e)$ of this function represents the utility, to some agent, of setting A to a particular value a (i.e., performing a particular action) when E takes a particular value e. The expected utility of an action a is defined as follows:

$$\mathbb{E}(u \mid a) = \sum_e p(e \mid do(a))\, u(a, e)$$

The expected utility of an action a, given an observation that the variables in $\boldsymbol{O}$ take the set of values $\boldsymbol{o}$, is given by the equation:

$$\mathbb{E}(u \mid a;\boldsymbol{o}) = \sum_e p(e \mid do(a), \boldsymbol{o})\, u(a, e)$$

where $p(e \mid do(a), \boldsymbol{o}) = \dfrac{p(e, \boldsymbol{o} \mid do(a))}{p(\boldsymbol{o} \mid do(a))}$. With these two equations in hand, we define the decision-theoretic value of the information contained in the observable variables $\boldsymbol{O}$ as follows:

$$VOI_u(\boldsymbol{O}) = \sum_o p(\boldsymbol{o})\, \underset{a}{max}\, \mathbb{E}(u \mid a;\boldsymbol{o}) - \underset{a}{max}\, \mathbb{E}(u \mid a)$$

That is, the value of the information contained in a set of observable variables is the average difference between the maximum utility an agent can expect when they have observed the value of all observable variables and the maximum utility that agent can expect when they have not made any observations. This is the standard decision-theoretic notion of value of information as defined by Blackwell (1953) and Good (1960).

Equipped with this definition, consider a case in which we move from a less compressed set of observable variables $\boldsymbol{O}$ (such as whether a medicine contains Type-1 Reptol, Type-2 Reptol, Type-1 Psylo, or Type-2 Psylo) to a more compressed set of observable variables $\widehat{\boldsymbol{O}}$ (such as whether a medicine contains Reptol or Psylo) for a fixed action variable A and an effect variable E. We can now calculate the decision-theoretic **value of lost information** (VOLI) with respect to this move, for an agent with utility function $u{:}R_A \times R_E \rightarrow \mathbb{R}$:

$$VOLI_u(\boldsymbol{O}, \widehat{\boldsymbol{O}}) = VOI_u(\boldsymbol{O}) - VOI_u(\widehat{\boldsymbol{O}})$$

This quantity tells us how costly the compression from $\boldsymbol{O}$ to $\widehat{\boldsymbol{O}}$ is for an agent whose preferences over values of A and E are given by the utility function u.

There is an important connection between our measure of the value of lost information and our measure of overall information loss. Specifically, for a natural formalization of an agent whose sole goal is to guess the correct distribution over an effect variable, the value of lost information in a compression is equal to the total amount of lost information involved in compression. To see this, consider an agent for whom the range of their action variable *A* consists of all possible probability distributions over an effect variable *E*, with each distribution in *A* representing the act of making a prediction about the probability of each value of *E* obtaining. The agent's utility function is such that for any action *a* and value *e* of *E*, $u(a,e) = \log_2 \frac{a(e)}{p(e)}$. This utility function rewards the agent for assigning high probability to *E* taking the value *e* when this actually occurs, with greater reward when said occurrence was unlikely according to the marginal distribution over *E*. Now consider a set of causal variables $\boldsymbol{C}$ with compression $\widehat{\boldsymbol{C}}$. For such an agent, $VOLI_u(\boldsymbol{C}, \widehat{\boldsymbol{C}}) = \mathscr{L}(\boldsymbol{C}, \widehat{\boldsymbol{C}}, E)$.[19] So, for the special case of an agent who only aims to make correct predictions about *E*, the value of lost information just is the total amount of information lost.

In this light, participants' responses in Experiments 1-3 can be interpreted as reflecting a trade-off between: i) satisfying the decision-theoretic goal of making correct predictions about *E*, and ii) maintaining a compressed causal representation of the system under study. However, in the following experiments we deliberately place agents in decision scenarios in which their goals involve more than solely guessing the correct distribution over the effect variable, to examine the more general role of the value of lost information in agents' evaluations of more and less compressed causal claims. Thus, what emerges from Experiments 1-4 is a general account

[19] To see this, consider the quantity $\sum_c q(c)\, max_{a \in A} \sum_e p(e|do(\boldsymbol{c}))\, u(a,e)$. The quantity $\sum_e p(e|do(\boldsymbol{c}))\, u(a,e)$ is maximized when $a(e) = p(e\,|\,do(\boldsymbol{c}))$ for all *e*. Thus, $\sum_c q(c)\, max_{a \in A} \sum_e p(e|do(\boldsymbol{c}))\, u(a,e) = = \sum_{c,e} q(\boldsymbol{c})\, p(e|do(\boldsymbol{c})) \log_2 \frac{p(e|do(\boldsymbol{c}))}{p(e)} = CME(\boldsymbol{C}, E)$. The quantity $max_a \sum_e p(e)\, u(a,e)$ is maximized at zero for $a(e) = p(e)$. Thus, $VOI_u(C) = CME(C,E) - 0 = CME(C,E)$. Repeating for $\widehat{\boldsymbol{C}}$ yields $VOI_u(\widehat{\boldsymbol{C}}) = CME(\widehat{\boldsymbol{C}}, E)$. Thus, $VOLI_u(C, \widehat{C}) = CME(\boldsymbol{C}, E) - CME(\widehat{\boldsymbol{C}}, E) = \mathscr{L}(C, \widehat{C}, E)$.

according to which people evaluating more and less compressed causal representations engage in a two-way tradeoff between compression and maintaining decision-theoretically valuable information, but where, in at least some cases, peoples' decision-theoretic goals include making accurate predictions (and therefore show sensitivity to information loss).

Motivation for Experiment 4

Experiment 4 tests **H3**: when an agent is placed in a decision context, that agent's evaluations of compressed causal claims will be sensitive to whether or not the value of the information lost in compression is zero or strictly positive. Specifically, we expect agents to evaluate compressed causal claims more positively when the value of the information lost in the relevant compression is zero.

One implication of these predictions is that agents' causal representations of their environments are primarily guided by their prudential *values*. That is, agents build causal models of their environments so as to achieve a representation that: i) allows for expected-utility-maximizing interventions on their environment, and ii) encodes observations that facilitate the choice of expected-utility-maximizing actions. In this respect, our hypothesis and the results supporting it are in keeping with work on value-guided task construal by Ho et al. (2022), as well as theoretical work by Brodu (2011), Kinney (2019), and Kinney and Watson (2020), arguing that prudential factors such as an agent's interest in realizing certain values of an effect variable and the value of the information provided by a causal variable determine the overall quality of compressed causal claims. However, Ho et al.'s experimental paradigms did not test the extent to which explicitly *causal* representations are value-guided, and so our experiments are also the first to test these ideas as descriptive claims about human evaluations of causal claims.

Experiment 4

In Experiment 4, we designed vignettes with a similar structure to those used in Experiments 1-3, but which also allowed us to manipulate whether the VOLI realized in the move from a less compressed to more compressed causal model of data presented in the vignette was zero or strictly positive. We hypothesized that when VOLI is zero, participants would tolerate information loss, and therefore rate causal claims embedded in a more compressed representation at least as highly as those embedded in a less compressed representation. By contrast, when VOLI is strictly positive, we hypothesized that participants would be less tolerant of information loss, since the more compressed representation fails to include information that is prudentially valuable to them, and they would therefore prefer those claims embedded in a less compressed representation. As in our earlier studies, we also hypothesized that these patterns of evaluation would hold regardless of whether compression was achieved by coarsening a causal variable or eliding a background condition.

Participants

Participants were 372 adults recruited via Prolific. An additional 408 participants were excluded for failing comprehension checks, and an additional 26 participants were excluded due to experimenter error.[20,21] The sample of participants was 49.2% female and 48.5% male, with an age range of 19-93 and a mean age of 42.

[20] A bug in our code meant that these 26 participants did not receive accurate feedback on the answers that they gave to multiple choice questions that were used to exclude participants from analysis. This resulted in slightly fewer data points being collected than were pre-registered. We exclude the data produced by these participants from our analysis below for the sake of accuracy, but note that all reported significant results still hold if the data from these participants is included.

[21] Given the large number of exclusions, we repeated key analyses including all participants. Patterns of findings were similar overall; see Supplementary Materials for details.

Materials and Procedures

We presented participants with the results of fictional experiments involving headache medications or training programs for new employees at a company. Participants were told that headache medications/training programs had to be shown to be effective in a certain percentage of people in order to be recommended for stocking on a pharmacy's shelves/being required for new employees. Participants were then asked to answer questions about the decisions they would make about different headache medicines or training programs, and then asked a second set of questions about their pattern of decision making with respect to the first set of questions.

To illustrate, participants shown the vignette about headache medicines were shown the data in Table 4. The value of *x* was varied between participants, as was whether the first or second set of facts about each medication was shown. Participants shown these data were then told that their manager had asked them to stock any medicine that was shown to reduce the severity of headaches in at least *y*% of patients, where $1 < y < x$. They were then asked to answer the following yes-or-no questions:

A1. Would you stock Medication A?

A2. Would you stock Medication B?

A3. Would you stock Medication C?

A4. Would you stock Medication D?

The correct answer to 1A is always 'Yes', and the correct answer to 3A and 4A is always 'No.' Whether the correct answer to 2A is 'Yes' or 'No' depends on whether *y* is less than or greater than 70. This was manipulated between participants, by setting *y=50* for some and *y=80* for others. Participants who answered these questions incorrectly were given a second chance to answer. If they answered incorrectly again, their data were excluded from analysis. Participants were then shown a summary of the correct answers to the first set of questions, and then asked to say whether the following statements, or analogous statements for those assigned to different conditions, were true or false:

> B1. Whenever a medication contains Reptol, you would stock it, regardless of whether it is Type-1 or Type-2.
>
> B2. Whenever a medication contains Reptol, you would stock it if it is Type-1 but not if it is Type-2.
>
> B3. Whenever a medication contains Psylo, you would *not* stock it, regardless of whether it is Type-1 or Type-2.
>
> B4. Whenever a medication contains Psylo, you would stock it if it is Type-1 but not if it is Type-2.

The statement B1 is true when $y \leq 70$ and false when $y > 70$, B2 is true when $y > 70$ and false when $y \leq 70$, B3 is always true, and B4 is always false. Participants who answered these questions incorrectly were given a second chance to answer. If they answered incorrectly again, their data were excluded from analysis.

Participants were then asked to evaluate, on a scale from -3 (very bad) to 3 (very good), how good it would be to include the following causal claims in a summary of the data prepared for a colleague:

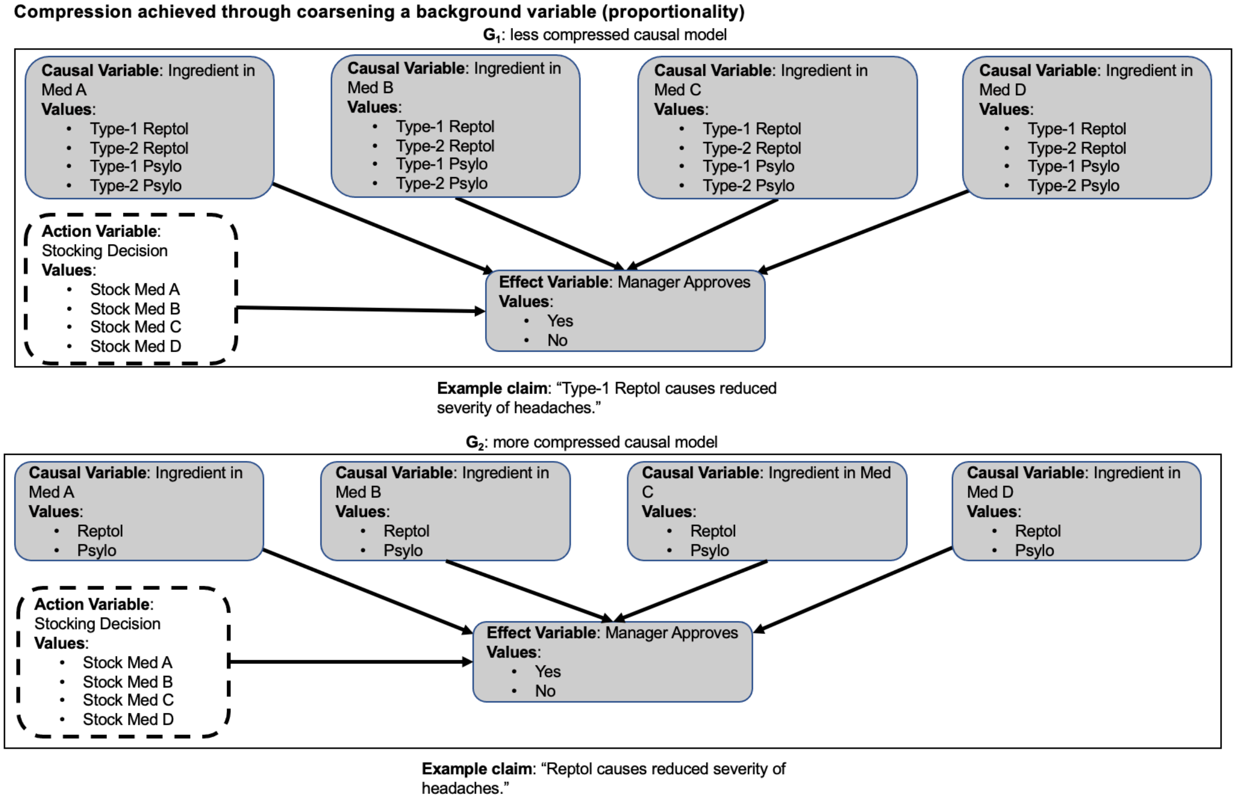
> *Compressed*: Reptol causes reductions in the severity of headaches.
>
> *High*: [Type-1 Reptol/Reptol taken with food] causes reductions in the severity of headaches.
>
> *Low*: [Type-2 Reptol/Reptol taken with food] causes reductions in the severity of headaches.
>
> *Compressed (Bad)*: Psylo causes reductions in the severity of headaches.
>
> *High (Bad)*: [Type-1 Psylo/Psylo taken with food] causes reductions in the severity of headaches.

Low (Bad): [Type-2 Psylo/Psylo taken with food] causes reductions in the severity of headaches.

See Figure 5 for a schematic representation of the compression inherent in endorsing the causal claim *Compressed* in the conditions in which compression involves coarsening a causal variable. As in Experiments 1-3, data from participants who assigned non-negative evaluations to the bad causal claims were excluded from analysis.

Figure 5

Graphs showing the Causal Relationships between Variables in Experiment 4


Compression achieved through coarsening a background variable (proportionality)
G_1: less compressed causal model
Causal Variable: Ingredient in Med A
Values:
Type-1 Reptol
Type-2 Reptol
Type-1 Psylo
Type-2 Psylo
Causal Variable: Ingredient in Med B
Values:
Type-1 Reptol
Type-2 Reptol
Type-1 Psylo
Type-2 Psylo
Causal Variable: Ingredient in Med C
Values:
Type-1 Reptol
Type-2 Reptol
Type-1 Psylo
Type-2 Psylo
Causal Variable: Ingredient in Med D
Values:
Type-1 Reptol
Type-2 Reptol
Type-1 Psylo
Type-2 Psylo
Action Variable: Stocking Decision
Values:
Stock Med A
Stock Med B
Stock Med C
Stock Med D
Effect Variable: Manager Approves
Values:
Yes
No
Example claim: "Type-1 Reptol causes reduced severity of headaches."
G_2: more compressed causal model
Causal Variable: Ingredient in Med A
Values:
Reptol
Psylo
Causal Variable: Ingredient in Med B
Values:
Reptol
Psylo
Causal Variable: Ingredient in Med C
Values:
Reptol
Psylo
Causal Variable: Ingredient in Med D
Values:
Reptol
Psylo
Action Variable: Stocking Decision
Values:
Stock Med A
Stock Med B
Stock Med C
Stock Med D
Effect Variable: Manager Approves
Values:
Yes
No
Example claim: "Reptol causes reduced severity of headaches."

Note: The top panel shows the uncompressed graph used in the proportionality condition (in which the claims *High* and *Low* are embedded), and the bottom panel shows the compressed graph used in the same condition (in which the claim *Compressed* is embedded). Whether the agent's manager approves of their choice of medication to stock depends on their choice of medication and the active ingredient in that medication, as indicated in the graph. The agent can observe the active ingredients in each medication, which is in turn informative about the likelihood that their manager approves their choice of medication.

Between participants, we manipulated:

- the vignette used (see Table 5 for a comparison of the pharmaceutical and employee training vignettes),
- the amount of information loss realized by the more compressed causal claim (by setting the value of *x* to either 85 or 98, resulting in information loss amounts of .01 and .06, respectively),
- whether compression was achieved by coarse-graining a variable, thus manipulating proportionality, or eliding a background variable, thus manipulating stability (this was done in the pharmaceutical vignette by showing participants either the first or second set of facts about each medication, respectively),
- whether the decision-theoretic value of the information lost in the move from a less compressed to a more compressed causal model was zero or strictly positive (by setting the value of *y* to either 50 or 80, respectively).

Table 5

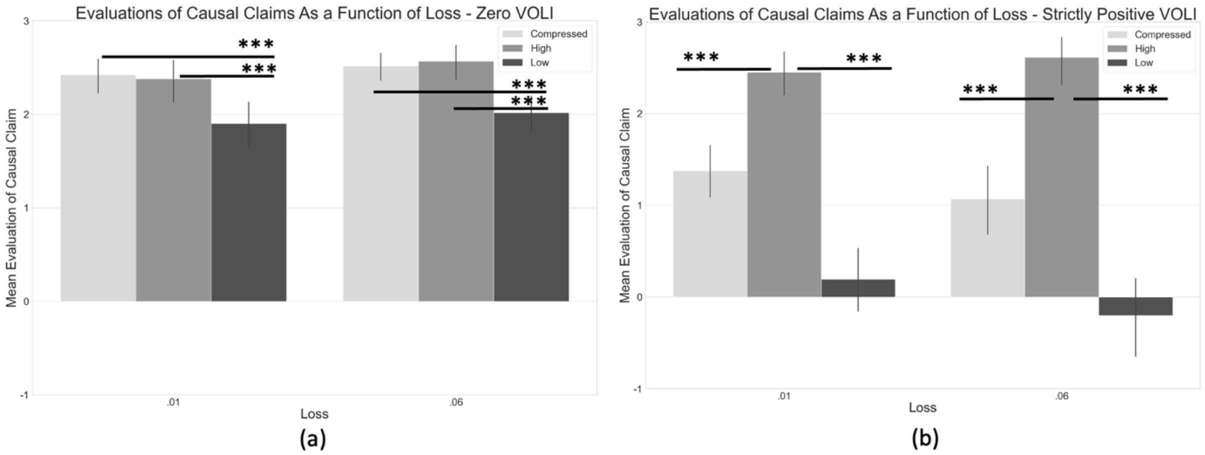
Structure of vignettes used in Experiment 4.

Vignette	Effect	Primary Cause	Secondary Cause	Background Condition
Pharma	Reduced Severity of headaches	Reptol/Psylo	Type-1/Type-2	Taken with/without food
Training	Becoming a successful employee	Case Studies/Simulations	Task-focused/Problem-focused	Held on weekends/weekdays

The final manipulation described above has no analog in Experiments 1-3, and so we explain it here in more detail. In the pharmaceutical case, when *y=50*, participants should recommend medications containing Reptol regardless of whether that Reptol is Type-1 or Type-2, and regardless of whether the medication must be taken with or without food. Under these conditions, the information lost in the compression is of no use to the agent's decision-making. Thus, VOLI in this case is zero. However, when *y=80*, only medications containing Type-1 Reptol/Reptol taken with food should be stocked on the shelves, and so the VOLI for the compression is strictly positive, on the assumption that agents assign strictly higher utility to complying with their manager's directions than to failing to comply. By manipulating whether VOLI was zero or strictly positive between participants, we were able to measure the extent to which the decision-theoretic *value* of the amount of information lost, controlling for the amount of information loss itself, influenced participants' evaluations.

Figure 6

Evaluations of Causal Claims by VOLI and Information Loss



Note: Bar plots showing mean evaluations in Experiment 4 (with 95% confidence intervals) of causal claims as a function of loss for participants assigned to the zero-VOLI condition (a) and the positive-VOLI condition (b). Triple asterisks indicate significant within-participants differences at the .001 level in a mixed effects ANOVA. Mixed ANOVA found that when the VOLI associated with compression is zero, there was no significant difference between Compressed and High ($\eta^2 = .001$, $p = .847$). There was a significant interaction effect between: i) the between-subjects difference in whether compression was achieved by coarsening a causal variable or removing a background variable, and ii) the within-subjects difference between evaluations of Compressed and High ($\eta^2 = .011$, $p = .017$). When VOLI is strictly positive, there was a significant difference between Compressed and High ($\eta^2 = .175$, $p < .001$). There was also a significant effect of the interaction between the amount of information lost due to compression and the vignette shown to participants ($\eta^2 = .017$, $p = .047$), and a significant interaction effect between the between-subjects difference in information loss due to compression and the within-subjects difference between evaluations of Compressed and High ($\eta^2 = .015$, $p = .005$).

Results

Figure 6 shows mean evaluations of each causal claim as a function of information loss under the different VOLI conditions. We first performed the same regressions as Experiments 1 and 2, with an additional binary independent variable added to represent whether the VOLI due to compression in the vignette shown to a participant was zero or strictly positive (once again, we also regressed our dependent variables on all possible interactions between all four of our independent variables). We found that COMPRESSED-HIGH, the difference between evaluations of *Compressed* and *High*, was significantly predicted by the following: i) whether VOLI was zero or strictly positive ($\beta = -.429$, $p = .001$, $R^2 = .242$), such that the difference between evaluations of *Compressed* and *High* was negligible when VOLI was zero, but much larger and negative (reflecting higher ratings for *High* than *Compressed*) when VOLI was strictly positive; ii) the amount of information loss due to compression ($\beta = -7.655$, $p = .010$, $R^2 = .242$), such that the extent to which ratings for *High* dominated ratings for *Compressed* increased as information loss increased; and iii) the interaction between whether VOLI was zero or strictly positive and the information loss due to compression ($\beta = -6.573$, $p = .027$, $R^2 = .242$), so that when VOLI was strictly positive, increases in information loss due to compression resulted in lower values of COMPRESSED-HIGH.

We followed up this interaction with independent tests of each VOLI condition. This revealed that when we restricted our analysis to just those cases in which VOLI is strictly positive, the sole significant predictor of COMPRESSED-HIGH is the amount of information lost in compression ($\beta = -11.809$, $p = .015$, $R^2 = .069$); by contrast, in those conditions for which VOLI was zero, none of the independent variables were significant predictors of COMPRESSED-HIGH. Finally, the original regression also revealed an interaction between: i) whether VOLI was zero or strictly positive, ii) the amount of information loss due to compression, and iii) the vignette shown to a participant ($\beta = -6.847, p = .021$, $R^2 = .242$). When VOLI was zero, participants who were shown the Pharma vignette tended to assign lower values to COMPRESSED-HIGH than those who were shown the training vignette. When VOLI was strictly positive, this relationship was reversed; participants who were shown the Pharma vignette tended to assign higher values to COMPRESSED-HIGH than those shown the Training vignette. As in Experiments 1-2, we found that the interaction between information loss due to compression, VOLI, and whether compression was achieved by coarsening a causal variable (proportionality) or removing a background variable (stability) was not a significant predictor of any of our dependent variables (COMPRESSED-HIGH: $\beta = -\ .200$, $p = .946$, $R^2 = .242$).

See the Supplemental Materials for the full pre-registered findings of Experiment 4.

Discussion

Experiment 4 found support for **H3**: When participants were asked to select a causal representation in the context of a particular decision problem, their tolerance for information loss was moderated by the decision-theoretic value of the information that was lost. Specifically, when the value of information lost in moving to a more compressed representation was zero, they rated the compressed claim (*Compressed*) and the less compressed claim associated with the highest probability (*High*) similarly, regardless of how much information loss was associated with the more compressed claim. But when the value of information lost through compression was strictly positive, participants favored the less compressed claim associated with the highest probability (*High*) over the more compressed claim (*Compressed*), and additionally showed sensitivity to the *amount* of information lost, with the compressed claim more strongly disfavored when it was associated with greater information loss. This suggests that agents do not merely consider whether lost information has strictly positive value or zero value when evaluating claims embedded in more or less compressed causal models, but instead continuously trade off an all-thing-considered preference for compression against the decision theoretic value of the information lost in compression, where there is an agential goal both to have enough information to accomplish a task and to be as predictively accurate as possible.

Regressions on the individual evaluations of each causal claim show a contrast between the effect of different amounts of information loss in Experiments 1 and 2 and the effect of different values for VOLI in Experiment 4. In Experiments 1 and 2, changes in the amount of information loss involved in compression mostly led to higher evaluations for *High*, without significant changes to evaluations of *Compressed*. By contrast, manipulations of VOLI in Experiment 4 largely led to changes in evaluations of *Compressed*, without significant changes in the value of *High*. This suggests that when more information is lost in compression, agents avoid that information loss by ensuring that they include detailed causal claims in their descriptions of a data-generating process. However, when the lost information is not decision-theoretically valuable to them, they do not downgrade their evaluations of more detailed causal claims, but instead upgrade their evaluations of causal claims embedded in a more compressed causal model.

One might worry that by only measuring participants' evaluations of causal claims, the preference we observe for more compressed causal explanations when information loss is low and VOLI is zero are entirely explained by pragmatic considerations along the lines of Grice's maxim of quantity (Grice, 1975), according to which a speaker should strive to be as informative as possible without providing any unnecessary information. In fact, we regard such maxims as highly amenable to our framework: much of what we have claimed can be seen as a formalization of Gricean ideas. Nonetheless, our own claims are more general insofar as they pertain to causal representation generally, not only interpersonal, linguistic communication. We therefore conducted an experiment, reported in the Supplemental Materials, in which participants saw the same stimuli as in Experiment 4, but in which they were asked to produce causal claims reflecting what they *learned*, rather than evaluate causal claims designed for communicating with others. To illustrate, participants shown the pharmaceutical vignette were required to describe, in at least 50 characters, "what you have learned about the efficacy of active ingredients in headache medicines." These participant-produced descriptions were then coded to assess the extent to which they expressed compressed representations of the data presented to the participant. This study found an effect of VOLI manipulations on participants' propensity to give compressed causal summaries of the data, in keeping with the findings of Experiment 4. For instance, when VOLI was zero, one participant wrote "Reptol reduces headache symptoms. Psylo does not. I would stock Reptol. I would not stock Psylo." When VOLI was strictly positive, one participant wrote: "the active ingredient Reptol Type-1 is most effective in reducing the severity of the patients' headaches." However, this study did not find an effect of information loss on participants' propensity to give compressed causal summaries of the data.

Nevertheless, one might still argue that the task in this follow-up experiment was still implicitly communicative, such that our results there, as in the other experiments reported herein, ultimately address the question of how people determine the level of compression with which they *talk about* the causal structure of the world, which is just one instance of the broader question of how people determine the level of compression with which they *represent* the causal structure of the world. Our aim in this paper is to address the latter question, and to examine how people determine the level of compression at which they represent the causal structure of the world. While we feel our dependent variables in Experiments 1-4, and their associated follow-ups, partially address the question of representation, it is nevertheless the case that they are communicative in nature. To this end, we conduct a fifth experiment in which we show that both the value and amount of the information lost in compression affect participants' judgments as to the optimal level of compression for a data set, where these judgments are measured in a way that does not require the participant to make or contemplate making an overtly communicative act. This fifth experiment is meant to rule out an interpretation of our results as being explained entirely by pragmatic norms of communication.

Experiment 5

Experiments 1-3 found that participants favored more compressed representations when the amount of information lost in compression was zero. This suggests that, all else being equal, participants assign some cost to storing a less compressed representation. Experiment 4 found that participants favored more compressed representations when the value of information lost in compression was zero. This suggests that the more valuable the information lost, the more willing participants should be to incur the representational costs of storing a less compressed representation. Schematically, this relationship between the cost of compression, on the one hand, and the value of lost information, on the other, can be written as follows:

$$\text{Propensity to Form Compressed Representation} \propto \text{Cost of Non-Compression} - \text{VOLI}$$

It follows from these observations that it should be possible to manipulate the propensity to form compressed representations not only by manipulating VOLI (as we do in Experiment 4), but also by manipulating cost. In Experiment 5, we do precisely this, and find that participants are more inclined to store a compressed representation of a data set when: i) compression is less costly, and ii) the amount (and therefore, the value) of the information lost in compression is lower. In contrast with Experiments 1-4, our key dependent variable does *not* involve communication on a participant's part. Rather, it directly measures the level of compression at which an agent chooses to store information about a data set.

Participants

Participants were 202 adults recruited via Prolific. An additional 29 participants were excluded for failing comprehension checks. The sample of participants was 50% female and 50% male, with an age range of 18-76 and a mean age of 36.

Materials and Procedures

Participants were told that they would be playing a role-playing game (RPG) in which their character began the game with 1000 health points and 1000 gold coins, and where the goal of the game was to end with as many health points and gold coins as possible. They were also told that over the course of the game, their character might sustain damage, reducing their total health points. To regain health points, they would be able to use gold coins to purchase remedies. Each remedy has a numerical strength, from 0-100, where the remedy's strength denotes the number of health points that can be regained by taking the remedy. To illustrate, if a participant's character has 800 health points after sustaining damage, and takes a remedy with a strength of 50, then the character will have 850 health points after taking the remedy.

Participants were shown a table with a list of remedies that they would potentially be able to buy, along with the strength of each remedy. The contents of each table depended on whether the participant had been randomly assigned to a "low loss" or "high loss" condition, and are shown in Table 6. The "low loss" condition is so-named because if this table were to be compressed into one that only compared the strength of mushrooms and flowers, less information about the differential strengths of remedies would be lost than if the same compression were performed in the "high loss" condition.

Table 6

Data shown to Participants in Experiment 5.

Low Loss		**High Loss**	
Remedy	**Strength**	**Remedy**	**Strength**
Green Mushroom	82	Green Mushroom	99
Yellow Mushroom	81	Yellow Mushroom	81
Purple Mushroom	80	Purple Mushroom	63
Yellow Flower	19	Yellow Flower	35
Red Flower	17	Red Flower	1

Participants were told that, later in the game, they would not have access to the information in the table they were shown, even though that information would be relevant. However, they would be able to use gold coins to purchase one of two note cards summarizing the data in the table. They would have access to the information on their purchased note card throughout the course of the game. Those two note cards, along with their costs, were as follows:

More Compressed: "Mushrooms are stronger than flowers." Cost: 100 Gold Coins.

Less Compressed: "Green mushrooms are strongest, followed by yellow mushrooms, then purple mushrooms, then yellow flowers, then red flowers." Cost: [100/120] Gold Coins.

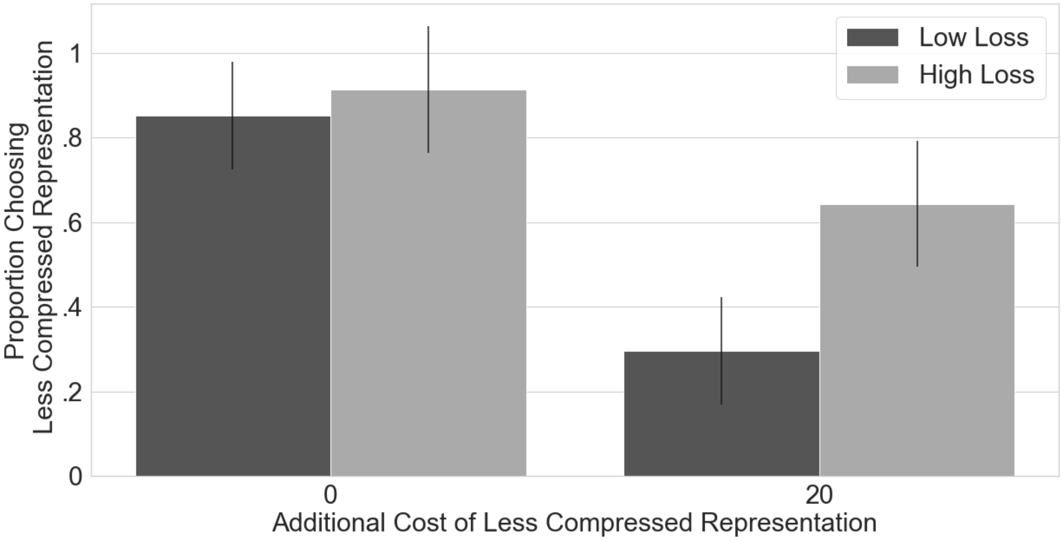
The cost of the less compressed note card was manipulated between participants, and set to either 100 or 120 gold coins, to reflect an additional cost of a less-compressed data representation of either 0 or 20. We predicted that, due to the increased loss of valuable information in the high loss category, participants would choose the less compressed causal representation more often in the high loss condition than in the low loss condition. We predicted further that this difference would be most pronounced when there is a positive cost to storing a less-compressed data representation; when storing less-compressed data comes with an explicit cost, agents will be more likely to incur that cost when doing so enables them to store a greater amount of valuable information.

In the remainder of the experiment, we told participants that their character had incurred damage, and then asked them to state how many gold coins they were willing to pay for a yellow mushroom to counteract that damage. They had access to the information on their purchased note card when making this decision. They were then asked to rate how helpful they found their note card, and then asked if they could recall the strength of each remedy from the table they were shown. These subsequent dependent variables were collected for exploratory purposes, but were not part of our core analysis, which concerns participants' choice of note card.

Results

Figure 7

Proportion of participants in Experiment 5 choosing a less compressed data representation, by level of information loss and the additional cost of a less compressed representation.



Note. Bars show 95% confidence intervals for proportions.

Figure 7 shows the proportion of participants selecting the less-compressed causal representation as a function of both the additional cost of the less compressed representation and the amount of valuable information lost in compression. As is clear from the figure, when there is no additional cost to storing a less compressed causal representation, a large majority of participants elect to do so. However, once there is a substantive cost to storing a less compressed causal representation of data, we see significant variation as to which participants are willing to select a less-compressed representation, with participants much more likely to do so when the amount of valuable information lost in compression is low.

We performed a binary logistic regression for a dependent variable representing whether or not a participant chose the less compressed data representation, regressing that variable against: i) the additional cost of the less compressed representation (Cost), and ii) a binary variable representing whether a participant was assigned to the low and high loss condition (Loss). The low loss condition was coded as -1 and the high loss condition was coded as 1. We found that both variables were significant predictors of whether or not a participant chose the less compressed representation (Cost: $\beta = -.116$, $p < .001$; Loss: $\beta = .602$, $p = .001$); participants with an additional cost of storing a less compressed representation were, all else being equal, less likely to do so, while participants in the high loss condition were, all else being equal, more likely to store a less compressed representation of the data, in keeping with our predictions. (In the Supplemental Materials, we report the results of another version of this experiment that replicates the effect of manipulating the cost of a less-compressed causal representation on participants' propensity to store such a representation, but does not find a significant effect of the overall value of the information lost in compression on participants' choice as to the level of compression at which they represent data.)

Discussion

The results of this experiment provide strong evidence that both the cost of storing a less compressed representation and the amount of valuable information lost in compression affect people's propensity to store more or less compressed representations of their environment. Unlike

Experiments 1-4, the results of Experiment 5 cannot be explained in terms of participants adhering to communicative norms, as their selection of a note card summarizing the data for use later in the game was not a directly communicative act, but was instead a choice about how to store information at a particular level of compression. One can view the choice of note card as a choice on the participant's part about how to represent the information in the data that they were shown within their extended mind (Clark & Chalmers, 1998), for subsequent use in navigating the environment of the role-playing game. Thus, the results of Experiment 5 speak in favor of our argument that agents choose causal representations by weighing a preference for compression against a desire to retain decision-theoretically relevant information.

The results of Experiment 5 also speak to a potential alternative explanation of the findings from Experiment 4. The alternative explanation is that our findings do not reflect our posited trade-off between compression and the value of information, but instead reflect the effects of adding a new feature (e.g., whether a medicine should be stocked) to a classification task. Specifically, Waldmann and Hagmayer (2006) show that participants presented with identical data about the causal capacities of exemplars nonetheless make different attributions of causal capacity depending on the categories they bring to the task and use to classify the exemplars. If the mechanisms identified by Waldmann and Hagmayer are applied to Experiment 4, then the four ingredients in the pharmaceutical example could be described as possessing the features shown in Table 7, where our manipulation of decision-theoretic threshold (50% vs. 80%) plays the role of an additional feature (stocked in the pharmacy or not) that shapes the categories participants use to classify exemplars. The two types of Reptol would then be grouped together when and because they both share the feature of being stocked at the pharmacy, rather than because of any effect of the change in value of the information lost in compression brought about by manipulating the decision-theoretic threshold. However, a similar analysis *cannot* be used to explain the results of Experiment 5, since the cost of a less-compressed representation is not a feature of any of the remedies that participants compress over (e.g., changing the cost of the more detailed note card from 100 gold coins to 120 gold coins does not change any features of a yellow mushroom).[22]

Table 7:

Putative features of the four causes in the pharmaceutical vignette of Experiment 4.

Ingredient	p(e|c) (i.e., Effectiveness)	Stocked at Pharmacy?
Type-1 Reptol	.85/.98	Yes
Type-2 Reptol	.7	Yes/No
Type-1 Psylo	.01	No
Type-2 Psylo	.01	No

General Discussion

As established in the introduction, representing a causal system involves a trade-off between informativeness and compression. How do agents manage this trade-off? In this paper we have put forward a theoretical framework, using the formal apparatus of Bayesian networks and information theory, that quantifies how much information is lost in the move from a less compressed causal representation of an agent’s environment to a more compressed representation of the same environment. We propose that agents trade off an all-things-considered preference for compression against a desire to avoid losing information due to compression. In so doing, we are also able to offer a unified account of the proportionality and stability of causal claims.

Experiments 1 and 2 support this part of our proposal. They show that when no information is lost in compression, people prefer causal claims embedded in more compressed models to those embedded in less compressed ones. When information loss due to compression is moderate, we do

[22] It is also worth noting that while the mechanism identified in Waldmann and Hagmayer (2006) could explain why participants were influenced by the threshold manipulation in Experiment 4, it is not clear how it could explain the interaction with information loss that was also observed.

not see a significant difference in peoples' evaluations of claims embedded in more and less compressed models. By contrast, when information loss due to compression is considerable, we see a strong preference for causal claims that are embedded in less compressed, more detailed causal models. Whether compression was achieved by coarsening a causal variable (proportionality) or removing a background condition (stability) did not make a significant difference with respect to participants' evaluations of causal claims, suggesting that our theoretical framework provides a unifying account of these important dimensions along which causal claims can be compared. Experiment 3 further corroborated this part of our proposal, while also showing that our account is able to explain results that are less well-explained in terms of causal contrast.

We further elaborated our framework by considering how decision-theoretic factors may influence agents' preferences over causal claims. Specifically, we introduced a framework for quantifying the value of the information lost in the move from a less compressed causal representation to a more compressed representation of the same environment, for an agent in a particular decision context. This allowed us to state precisely whether the information lost in a given compression is or is not valuable to a particular agent, in keeping with the broader theory that an agent's construal of the causal structure of their environment is fundamentally informed by the prudential values of that agent. We tested this aspect of our framework in Experiment 4, which found that when participants evaluate causal claims, they engage in a trade-off between a preference for compression and a preference for *valuable* information, where the value of information is determined by the decision-theoretic context in which a particular data-generating process is presented to participants. Importantly, this preference for valuable information is continuous; the greater the amount of valuable information lost in a compression, the more negatively participants evaluate a compressed causal claim. In keeping with our hypothesis, discussed in the introduction, that human agents are capable of selecting from among different possible causal models of their environment to fit a specific context, the results of Experiment 4 suggest that this selection process is strongly influenced by the decision-theoretic structure of a given context in which a mental causal model is deployed.

Finally, we noted that, while we took our dependent variables in Experiments 1-4 to measure the extent to which participants were representing their environment at a given level of compression, they may be alternatively interpreted as reflecting solely communicative norms. To bolster the case for a representational interpretation of our findings, we conducted Experiment 5, which measured the effect of both the cost of a less compressed representation and the value of the information lost in compression on a variable that is much more plausibly interpreted as measuring participants' representational preferences than their communicative ones. This experiment found a significant effect of both the cost of a less compressed representation and the value of the information lost on participants' propensity to represent data at a certain level of compression.

The overall takeaway from our five experiments is as follows. When agents represent the causal structure of their environment, they have an extremely wide latitude with respect to how compressed that representation should be. Ultimately, the level of compression that an agent chooses for such a representation is determined by a trade-off wherein agents seek to minimize the loss of valuable information while maximizing compression. We note that in the follow-ups to Experiments 4 and 5 reported in the Supplemental Materials, we do not see a significant effect of the *amount* of information lost in compression on participants' preferences as to the level of compression at which they represent data. Thus, based on the data collected here, we take it to be something of an open question whether, when a particular decision context is specified, minimizing the loss of information involves a continuous preference for less information loss when the information loss is valuable, or a categorical preference for representations in which the value of lost information is zero and against representations in which the value of lost information is positive. (We speculate that information loss is likely to play *some* role even when the value of information is zero, given both the results of Experiments 4 and 5 and the fact that agents may have uncertainty regarding their own prudential values, or anticipate that they might change.) What *is* clear is that the tradeoff between informativeness and compression holds independently of whether compression is achieved by coarsening a causal variable or eliding a background condition. This suggests a common framework for understanding these two salient varieties of compression.

Finally, it is worth noting that while our discussion here pertains to explicitly *causal* representations of data-generating processes, it may be extended further to include non-causal predictive models that might be used in agential representation of the environment and cognition. As long as these models use random variables and involve predicting a particular outcome of interest, much of the formal apparatus and experimental paradigms developed here will remain applicable. Thus, we hope that our results here have provided a general framework for thinking through the relationship between information loss, compression, and prudential values as the chief drivers of the processes whereby humans and other agents construct formal representations of their environment, for use in both navigating and intervening upon the world in which they live.

Relationship to Existing Model Selection Techniques

In statistics, the tradeoff between model simplicity and informativeness is most commonly understood through the lens of various "information criteria" used for model selection (Akaike, 1998; Schwarz, 1978). Indeed, there are similarities between our discussion of causal model selection and the much larger statistical literature on model selection via information criteria, in which the fundamental tradeoff is between selecting a model that captures all the data and a model that contains a small number of parameters. Nevertheless, there are also important differences. First, when we coarse-grain a causal variable in a model and leave the rest of the model unchanged, we do not necessarily change the number of statistical parameters in the model (e.g., the probability distribution over a continuously-valued variable may have the same number of parameters as a distribution over a discretization of that variable), though such coarsening may change the parameters that maximize the likelihood of the data. Nevertheless, in our framework a model with a more coarse-grained variable is considered to be more compressed, all else being equal, than the same model with a more fine-grained version of the same variable. In addition, we trade off compression or simplicity against the amount of information shared between causes and their effects, rather than the likelihood of a model given some set of observed data. Our measure is meant to be applied in cases where more and less compressed models of some system are all well-supported by data, and yet there is nevertheless a trade-off between the simplicity of a model and the informativeness of cause-effect relationships within that model. That said, there are likely conditions under which our framework and other statistical techniques for model selection will make similar recommendations. We leave it to future work to examine these conditions in depth.

Limits of the Current Study

While we take our experimental results to confirm the theoretical framework presented, we acknowledge that they have some important limitations. As noted at the outset, we focus on qualitative predictions derived from principled measures of compression and information loss, but we do not claim that our specific measures correspond to the algorithms by which humans compute these quantities. This leaves it to future work to develop and test quantitative process models.

Additionally, our experiments were conducted solely on U.S.-based participants recruited through on-line platforms, and so we are limited in the extent to which we can generalize to all agents faced with the task of creating causal models of the world. In particular, it is a largely open question whether individual scientists or groups of scientists would instantiate the same trade-offs observed here. In addition, our results are constrained to relatively simple causal scenarios that can be presented and understood in a matter of minutes, such that our results are of less significance in understanding deliberative, detailed causal understanding. This is a notable limitation if – as seems likely – variable choice interacts with information search in a dynamic process of inquiry, whereby current representations guide interventions, which in turn generate the data that revises representations. Investigating this interactive process is an important direction for future research.

Directions for Future Work

The current results suggest several intriguing avenues for future work that would generalize our findings to other areas of psychology. One such avenue emerges when we consider that the human capacity for compressed representation of causal structure begins very early in life. Future work in developmental psychology could show whether the process of selecting compressed representations of formal structure is fundamentally goal-oriented in very young humans. If this is the case, it would lend further support to the theory that our early-emerging and core commitments regarding the causal structure of data-generating processes are shaped by our pragmatic goals as agents.

Another intriguing line for future research concerns the psychology of social categorization. When we group people into categories such as race or gender, we situate them within a socially-constructed causal nexus based on group stereotypes. Our choices of social categories used to classify people are extremely ethically fraught, such that understanding why we group people in the particular ways that we do is a central goal of social psychology. If we are correct in thinking that the classification schema used in causal reasoning generally are downstream of our prudential goals as agents, then work in social psychology might establish that the same holds for social classification. This could yield a new analysis of the ethics of stereotyping, according to which the moral valence of a particular classification schema for individuals is tied to the moral valence of the prudential goals that led to that schema.

On the formal and mathematical side, as acknowledged above, our measure of information loss is an application of rate distortion theories developed in other areas of cognitive science. In rate distortion theories, one often finds a setting of parameters such that agents' preferences over more and less distorted or compressed information channels follow a "rate distortion curve" showing the optimal level of distortion (Zaslavsky et al., 2018). A potentially fruitful formal project would involve spelling out, in full formal detail, how our measure of information loss can be re-stated as a rate distortion curve, with the decision-theoretic value of information being used to set key parameters that determine the shape of the curve. Such a formal study would amount to a significant unification of the literature in rate distortion theory and the literature on causal variable choice.

These potential directions for future research, alongside our analysis of the results of the current studies, speak to the fruitfulness of our theoretical approach that combines information theory and decision theory to offer a unified analysis of causal structure selection in cognition. We believe that future applications of this approach may lead to a more comprehensive understanding of the relationship between goal-setting, planning, and representation of the environment in humans, non-human animals, and artificially intelligent agents.

Conclusion

We have proposed a theoretical framework for measuring the amount of information lost in the move from a less-compressed to a more-compressed causal model of an environment. This framework allows us to give a unified account of the proportionality and the stability of causal claims. This framework can additionally quantify and incorporate the decision-theoretic *value* of the information that is lost in compression. Over the course of four experiments, we demonstrated the empirical adequacy of this framework in predicting people's evaluation and generation of causal claims. This suggests that, as per our hypothesis, human representations of the causal structure of the environment do trade-off valuable information against compression in a context-dependent way.

References

Akaike, H. (1998). Information theory and an extension of the maximum likelihood principle. In *Selected papers of hirotugu akaike* (pp. 199–213). Springer.

Aronowitz, S., & Lombrozo, T. (2020). Experiential Explanation. *Topics in Cognitive Science*, *12*(4), 1321–1336. https://doi.org/10.1111/tops.12445

Ay, N., & Polani, D. (2008). Information flows in causal networks. *Advances in Complex Systems*, *11*(01), 17–41.

Bechlivanidis, C., Lagnado, D. A., Zemla, J. C., & Sloman, S. (2017). Concreteness and abstraction in everyday explanation. *Psychonomic Bulletin & Review*, *24*(5), 1451–1464. https://doi.org/10.3758/s13423-017-1299-3

Beckers, S., & Halpern, J. Y. (2019). Abstracting causal models. *Proceedings of the Aaai Conference on Artificial Intelligence*, *33*(01), 2678–2685.

Blackwell, D. (1953). Equivalent comparisons of experiments. *The Annals of Mathematical Statistics*, 265–272.

Bourrat, P. (2021a). Heritability, causal influence and locality. *Synthese*, *198*(7), 6689–6715.

Bourrat, P. (2021b). Measuring Causal Invariance Formally. *Entropy*, *23*(6), 690.

Bramley, N. R., Dayan, P., Griffiths, T. L., & Lagnado, D. A. (2017). Formalizing Neurath's ship: Approximate algorithms for online causal learning. *Psychological Review*, *124*(3), 301.

Bramley, N. R., Lagnado, D. A., & Speekenbrink, M. (2015). Conservative forgetful scholars: How people learn causal structure through sequences of interventions. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *41*(3), 708.

Brodu, N. (2011). Reconstruction of epsilon-machines in predictive frameworks and decisional states. *Advances in Complex Systems*, *14*(05), 761–794.

Buchsbaum, D., Griffiths, T. L., Plunkett, D., Gopnik, A., & Baldwin, D. (2015). Inferring action structure and causal relationships in continuous sequences of human action. *Cognitive Psychology*, *76*, 30–77.

Chater, N., & Vitányi, P. (2003). Simplicity: A unifying principle in cognitive science? *Trends in Cognitive Sciences*, *7*(1), 19–22.

Cheng, P. W. (1997). From covariation to causation: A causal power theory. *Psychological Review*, *104*(2), 367.

Clark, A., & Chalmers, D. (1998). The extended mind. *Analysis*, *58*(1), 7–19.

DiMarco, M. (2021). Wishful Intelligibility, Black Boxes, and Epidemiological Explanation. *Philosophy of Science*, *88*(5), 824–834.

Fauconnier, G., & Turner, M. (2008). *The way we think: Conceptual blending and the mind's hidden complexities*. Basic books.

Franklin-Hall, L. R. (2016). High-level explanation and the interventionist's 'variables problem.' *The British Journal for the Philosophy of Science*, *67*(2), 553–577.

Gebharter, A., & Eronen, M. I. (2021). Quantifying proportionality and the limits of higher-level causation and explanation. *The British Journal for the Philosophy of Science*, 714818. https://doi.org/10.1086/714818

Gerstenberg, T., Goodman, N. D., Lagnado, D. A., & Tenenbaum, J. B. (2021). A counterfactual simulation model of causal judgments for physical events. *Psychological Review*, *128*(5), 936.

Good, I. J. (1960). The paradox of confirmation. *The British Journal for the Philosophy of Science*, *11*(42), 145–149.

Gopnik, A., & Sobel, D. M. (2000). Detecting blickets: How young children use information about novel causal powers in categorization and induction. *Child Development*, *71*(5), 1205–1222.

Gopnik, A., & Tenenbaum, J. B. (2007). Bayesian networks, Bayesian learning and cognitive development. In *Developmental science* (Vol. 10, Issue 3, pp. 281–287). Citeseer.

Grice, H. P. (1975). Logic and conversation. In *Speech acts* (pp. 41–58). Brill.

Griffiths, P. E., Pocheville, A., Calcott, B., Stotz, K., Kim, H., & Knight, R. (2015). Measuring causal specificity. *Philosophy of Science*, *82*(4), 529–555.

Griffiths, T. L., Kemp, C., & Tenenbaum, J. B. (2008). *Bayesian models of cognition*.

Harbecke, J. (2021). Causal Proportionality as an Ontic and Epistemic Concept. *Erkenntnis*, 1–23.

Ho, M. K., Abel, D., Correa, C. G., Littman, M. L., Cohen, J. D., & Griffiths, T. L. (2022). People construct simplified mental representations to plan. *Nature*, *606*(7912), Article 7912. https://doi.org/10.1038/s41586-022-04743-9

Hoel, E. P. (2017). When the map is better than the territory. *Entropy*, *19*(5), 188.

Hoffmann-Kolss, V. (2014). Interventionism and higher-level causation. *International Studies in the Philosophy of Science*, *28*(1), 49–64.

Icard, T. F., Kominsky, J. F., & Knobe, J. (2017). Normality and actual causal strength. *Cognition*, *161*, 80–93.

Keil, F. C. (2006). Explanation and understanding. *Annu. Rev. Psychol.*, *57*, 227–254.

Kinney, D. (2019). On the explanatory depth and pragmatic value of coarse-grained, probabilistic, causal explanations. *Philosophy of Science*, *86*(1), 145–167.

Kinney, D., & Watson, D. (2020). Causal feature learning for utility-maximizing agents. *International Conference on Probabilistic Graphical Models*, 257–268.

Kirfel, L., Icard, T., & Gerstenberg, T. (2021). Inference from explanation. *Journal of Experimental Psychology: General*.

Korb, K. B., Nyberg, E., & Hope, L. R. (2011). A new causal power theory. In *Causality in the Sciences* (pp. 628–652). Oxford University Press.

Lien, Y., & Cheng, P. W. (2000). Distinguishing genuine from spurious causes: A coherence hypothesis. *Cognitive Psychology*, *40*(2), 87–137.

List, C., & Menzies, P. (2009). Nonreductive physicalism and the limits of the exclusion principle. *The Journal of Philosophy*, *106*(9), 475–502.

Lombrozo, T. (2007). Simplicity and probability in causal explanation. *Cognitive Psychology*, *55*(3), 232–257.

Lombrozo, T. (2010). Causal–explanatory pluralism: How intentions, functions, and mechanisms influence causal ascriptions. *Cognitive Psychology*, *61*(4), 303–332.

Marzen, S. E., & DeDeo, S. (2017). The evolution of lossy compression. *Journal of The Royal Society Interface*, *14*(130), 20170166.

Morris, A., Phillips, J. S., Icard, T., Knobe, J., Gerstenberg, T., & Cushman, F. (2018). *Causal judgments approximate the effectiveness of future interventions*.

Murphy, G. (2004). *The big book of concepts*. MIT press.

O'Neill, K., Henne, P., Bello, P., Pearson, J., & De Brigard, F. (2021). *Degrading causation*.

O'Neill, K., Henne, P., Bello, P., Pearson, J., & De Brigard, F. (2022). Confidence and gradation in causal judgment. *Cognition*, *223*, 105036.

Pacer, M., & Lombrozo, T. (2017). Ockham's razor cuts to the root: Simplicity in causal explanation. *Journal of Experimental Psychology: General*, *146*(12), 1761.

Papineau, D. (2022). The statistical nature of causation. *The Monist*, *105*(2), 247–275.

Pearl, J. (1994). A probabilistic calculus of actions. In *Uncertainty Proceedings 1994* (pp. 454–462). Elsevier.

Pearl, J. (2000). *Causality: Models, Reasoning and Inference*. Cambridge University Press.

Pocheville, A., Griffiths, P. E., & Stotz, K. (2017). Comparing causes—An information-theoretic approach to specificity, proportionality and stability. *Proceedings of the 15th Congress of Logic, Methodology and Philosophy of Science*, 93–102.

Quillien, T. (2020). When do we think that X caused Y? *Cognition*, *205*, 104410.

Rosch, E. (1978). *Principles of categorization*.

Ross, L. N. (2015). *Causal control: A rationale for causal selection*.

Schwarz, G. (1978). Estimating the dimension of a model. *The Annals of Statistics*, 461–464.

Sims, C. R. (2016). Rate–distortion theory and human perception. *Cognition*, *152*, 181–198.

Spellman, B. A. (1997). Crediting causality. *Journal of Experimental Psychology: General*, *126*(4), 323.

Spirtes, P., Glymour, C., & Scheines, R. (2000). Causation, Prediction, and Search. *MIT Press Books*, *1*.

Tishby, N., Pereira, F. C., & Bialek, W. (2000). The information bottleneck method. *ArXiv Preprint Physics/0004057*.

Vasilyeva, N., Blanchard, T., & Lombrozo, T. (2018). Stable causal relationships are better causal relationships. *Cognitive Science*, *42*(4), 1265–1296.

Waldmann, M. R., & Hagmayer, Y. (2006). Categories and causality: The neglected direction. *Cognitive Psychology*, *53*(1), 27–58.

Watson, D., & Silva, R. (2022). *Causal discovery under a confounder blanket* (arXiv:2205.05715). arXiv. http://arxiv.org/abs/2205.05715

Weslake, B. (2013). Proportionality, contrast and explanation. *Australasian Journal of Philosophy*, *91*(4), 785–797.

Wilkenfeld, D. A. (2019). Understanding as compression. *Philosophical Studies*, *176*(10), 2807–2831.

Wojtowicz, Z., Chater, N., & Loewenstein, G. (2021). *The motivational processes of sense-making*.

Woodward, J. (2008). Mental causation and neural mechanisms. *Being Reduced: New Essays on Reduction, Explanation, and Causation*, 218–262.

Woodward, J. (2010). Causation in biology: Stability, specificity, and the choice of levels of explanation. *Biology & Philosophy*, *25*(3), 287–318.

Woodward, J. (2016a). Causal cognition: Physical connections, proportionality, and the role of normative theory. *Of Psychology: Causality and Psychological Subject*, 105.

Woodward, J. (2016b). The problem of variable choice. *Synthese*, *193*(4), 1047–1072.

Woodward, J. (2019). On Wolfgang Spohn's Laws of Belief. *Philosophy of Science*, *86*(4), 759–772.

Woodward, J. (2021a). *Causation with a human face: Normative theory and descriptive psychology*. Oxford University Press.

Woodward, J. (2021b). Explanatory autonomy: The role of proportionality, stability, and conditional irrelevance. *Synthese*, *198*(1), 237–265.

Yablo, S. (1992). Mental causation. *The Philosophical Review*, *101*(2), 245–280.

Zaslavsky, N., Kemp, C., Regier, T., & Tishby, N. (2018). Efficient compression in color naming and its evolution. *Proceedings of the National Academy of Sciences*, *115*(31), 7937–7942.

Appendix A

In this appendix, we provide our full formal framework for measuring the amount of information that is lost in the move from one Bayesian network representing the causal structure of some system to a second Bayesian network, where this second Bayesian network amounts to a more compressed representation of a given target system than the first. This allows us to state precisely our claim that both the proportionality and stability of a causal claim can be defined in terms of information loss between causal models. It also provides the theoretical framework within which our first three experimental paradigms are situated.

Variables, Coarsenings, and Causal Bayesian Networks

We begin with a **probability space**, which is a triple (Ω, Σ, p) in which Ω is a **sample space** of primitive possibilities (i.e., a set of possible worlds), Σ is an **algebra** on Ω (i.e., a set of subsets of Ω that is closed under union, complement, and intersection), and p is a **probability distribution** on Σ that satisfies the Kolmogorov axioms. A **random variable** $X\colon \Omega \rightarrow R_X$ is function from the sample space into some set R_X (i.e., the **range** of the random variable). As stated in the introduction, in this paper we assume that random variables are surjective but not injective functions on the sample space, meaning that multiple possible worlds are often mapped to the same value of a random variable. This clarifies one sense in which representations that use random variables are compressions of their targets; they clump together many possible observations under a single label. A random variable is said to be **measurable** with respect to a probability space (Ω, Σ, p) if and only if for any $x \in R_X$, $X^{-1}(x) \in \Sigma$. This allows us to assign a probability to the event that the variable *X* takes the value *x*, using the equation $p(X = x) = p\left(X^{-1}(x)\right)$.

For any random variable *X* that is measurable with respect to a probability space (Ω, Σ, p) let $\sim_X$ be an equivalence relation defined on Ω such that $\omega \sim_X \omega'$ if and only if $X(\omega) = X(\omega')$. A random variable $\hat{X}$ is a **coarsening** of *X* if and only if, for any $\omega, \omega' \in \Omega$: i) if $\omega \sim_X \omega'$ then $\omega \sim_{\hat{X}} \omega'$, and ii) there exists an $\omega, \omega' \in \Omega$ such that $\omega \sim_{\hat{X}} \omega'$ and $\omega \nsim_X \omega'$. If $\hat{X}$ is a coarsening of *X*, then *X* is a **refinement** of $\hat{X}$. The definition of coarsening captures the intuitive idea that coarser-grained random variables define a more general compression of the possibility space on which they are defined than their more fine-grained counterparts. That is, all possibilities treated as equivalent by a random variable *X* are also treated as equivalent by its coarsening $\hat{X}$, but some possibilities treated as equivalent by $\hat{X}$ are not treated as equivalent by *X*.

Moving to the use of Bayesian networks to represent causal structure, let $\mathscr{V}_{\mathscr{P}}$ be a set of random variables that are each measurable with respect to a probability space $\mathscr{P} = (\Omega, \Sigma, p)$. Let $\mathscr{E}_{\mathscr{V}_{\mathscr{P}}}$ be an acyclic set of ordered pairs, or edges, relating the variables in $\mathscr{V}_{\mathscr{P}}$. These are represented pictorially as arrows in the causal graph depicting relations of direct causation from one variable to another. A **causal Bayes net** $\mathscr{G}_{\mathscr{P}}$ is a pair ($\mathscr{V}_{\mathscr{P}}$, $\mathscr{E}_{\mathscr{V}_{\mathscr{P}}}$) that satisfies the following conditions:

1. According to the probability distribution p in the probability space $\mathscr{P}$ with respect to which the Bayes net is defined, all variables are independent of their non-descendants, conditional on their parents (**Markov condition**).
2. There is no set of edges $\mathscr{E}^{*}_{\mathscr{V}_{\mathscr{P}}} \subset \mathscr{E}_{\mathscr{V}_{\mathscr{P}}}$ such that ($\mathscr{V}_{\mathscr{P}}$, $\mathscr{E}^{*}_{\mathscr{V}_{\mathscr{P}}}$) satisfies the Markov condition (**Minimality condition)**.
3. No element of $\mathscr{V}_{\mathscr{P}}$ is a coarsening of or identical to any other element of $\mathscr{V}_{\mathscr{P}}$ (**Co-possibility condition)**.

The Markov and Minimality conditions formalize the idea that the value of each variable in a causal Bayes net is determined by all and only its parents (i.e., its direct causes), plus an exogenous source of error not accounted for in the Bayes net and not correlated with the error in any other variables. The Co-possibility condition ensures that all functional relationships between variables in a causal Bayes net are indeed causal, rather than logical, in nature.

An important feature of a causal Bayes net is that it allows us to calculate the probability distribution over the variables in the Bayes net given one or more hypothetical interventions setting the value(s) of variables in the Bayes net, in keeping with the “do-calculus” of (Pearl, 2000). For any given causal Bayes net $\mathscr{G}_{\mathscr{P}}$, we can calculate the probability distribution over any variable V in the set $\mathscr{V}_{\mathscr{P}}$, given an intervention setting some set of variables X to some set of values x, using the following formula:

$$p_{\mathcal{G}_{\mathcal{P}}}(v \mid do(x)) = \begin{cases} p\left(v \,\middle|\, par_{\mathcal{G}_{\mathcal{P}}}(V)\right) if\ V \notin X \\ 1 if\ V \in X\ and\ v\ is\ consistent\ with\ x \\ 0\ otherwise \end{cases}$$

where $\mathsf{par}_{\mathcal{G}_{\mathcal{P}}}(V)$ denotes the values taken by the parents of V in $\mathcal{G}_{\mathcal{P}}$. The idea here is that when the variable that the distribution is defined over is not intervened upon, the distribution is determined solely by the value taken by the parents of that variable. In practice, these values are not always known, but may be known if the variable(s) intervened upon include parents of the variable over which the distribution is defined. Where parents are not known, they are marginalized over. This allows us to derive the probability distribution that would be defined over any variable in the causal Bayes net, if any other variable in the same causal Bayes net were set to some value via an exogenous, “surgical" intervention on the data-generating system.

For any given causal Bayes net, we can define an equivalence relation $\sim_{\mathcal{V}_{\mathcal{P}}}$ on the sample space Ω, such that $\omega \sim_{\mathcal{V}_{\mathcal{P}}} \omega'$ if and only if for all $V \in \mathcal{V}_{\mathcal{P}}$, $V(\omega) = V(\omega')$. A causal Bayes net $\mathcal{G}_{\mathcal{P}} = \left(\mathcal{V}_{\mathcal{P}}, \mathcal{G}_{\mathcal{V}_{\mathcal{P}}}\right)$ is a **more compressed** representation of a given target process than an alternative causal Bayes net $\mathcal{G}'_{\mathcal{P}} = \left(\mathcal{V}'_{\mathcal{P}}, \mathcal{G}_{\mathcal{V}'_{\mathcal{P}}}\right)$ if and only if: i) for any $\omega, \omega' \in \Omega$, if $\omega \sim_{\mathcal{V}'_{\mathcal{P}}} \omega'$, then $\omega \sim_{\mathcal{V}_{\mathcal{P}}} \omega'$and ii) there exists an $\omega, \omega' \in \Omega$, such that $\omega \sim_{\mathcal{V}_{\mathcal{P}}} \omega'$ but $\omega \nsim_{\mathcal{V}'_{\mathcal{P}}} \omega'$. Thus, in an analogy to the coarsening-of relation between variables, a more compressed Bayes net defines a more general equivalence relation over the sample space than a less compressed Bayes net defined over the same sample space, and representing the same underlying dynamics.

Measuring Information Loss

We are now in a position to introduce a formal measure of the amount of information that is lost about an effect of interest E, with respect to some sets of causal variables of interest $\boldsymbol{C}$ and $\boldsymbol{C}'$, when we move from one Bayes net $\mathcal{G}_{\mathcal{P}}$ to a more compressed Bayes net $\mathcal{G}_{\mathcal{P}}^{'}$ representing the same data-generating process. Specifically, we define an information loss function $\mathcal{L}\left(\mathcal{G}_{\mathcal{P}},\mathcal{G}'_{\mathcal{P}},\boldsymbol{C},\boldsymbol{C}',E,q\right)$ as follows:

$$\mathcal{L}\left(\mathcal{G}_{\mathcal{P}},\mathcal{G}'_{\mathcal{P}},\boldsymbol{C},\boldsymbol{C}',E,q\right)=\sum_{c}q(c)\sum_{e}p_{\mathcal{G}_{\mathcal{P}}}(e\mid do(c))\log_2\frac{p_{\mathcal{G}_{\mathcal{P}}}(e\mid do(c))}{p(e)}-\sum_{\boldsymbol{c}'}q(\boldsymbol{c}')\sum_{\boldsymbol{e}}p_{\mathcal{G}'_{\mathcal{P}}}(e\mid do(\boldsymbol{c}'))\log_2\frac{p_{\mathcal{G}'_{\mathcal{P}}}(e\mid do(\boldsymbol{c}'))}{p(e)}$$

The probabilities $q(\boldsymbol{c})$ and $q(\boldsymbol{c}')$ are respectively interpreted as the probability of an intervention setting the variables in $\boldsymbol{C}$ to the vector of values $\boldsymbol{c}$ and the probability of setting the variables in $\boldsymbol{C}'$ to the vector of values $\boldsymbol{c}'$.

Measuring Proportionality Using Information Loss

Recall that one way of moving from a causal Bayes net $\mathcal{G}_{\mathcal{P}}$ to a more compressed Bayes net $\mathcal{G}'_{\mathcal{P}}$ is by replacing a variable *C* in the graph $\mathcal{G}_{\mathcal{P}}$ with its coarsening $\widehat{C}$, and leaving all other variables unchanged. By measuring the amount of information that is lost in the move from $\mathcal{G}_{\mathcal{P}}$ to $\mathcal{G}'_{\mathcal{P}}$, we can compare the amount of information that *C* communicates about some effect variable *E* to the amount of information that $\widehat{C}$ communicates about the same variable *E*, thereby comparing the causal claims '*C* causes *E*' and '$\widehat{C}$ causes *E*' with respect to their proportionality.

More precisely, let $G_{\mathcal{P}}=\left(\mathcal{G}^1_{\mathcal{P}},\ldots,\mathcal{G}^n_{\mathcal{P}}\right)$ be a sequence of causal Bayes nets, such that the only difference between two causal Bayes nets $\mathcal{G}^i_{\mathcal{P}}$ and $\mathcal{G}^{i+1}_{\mathcal{P}}$ is the replacement of a single variable with a coarsening thereof.[23] This yields a sequence of variables $C=\left(C_1,\ldots,C_n\right)$, with each C_i a variable in the causal Bayes net $\mathcal{G}^i_{\mathcal{P}}$ and a coarsening of all variables $C_{j<i}$. We then say that, in the context of a such a sequence, a variable C_i is **proportional** with respect to an effect variable *E* to the extent that $\mathcal{L}\left(\mathcal{G}^j_{\mathcal{P}},\mathcal{G}^i_{\mathcal{P}},\{C_j\},\{C_i\},E,q\right)$ is relatively small for all $j<i$. That is, proportional choices

[23] Formally, for any $i<n$ there is a bijection $f{:}\mathcal{V}^i_{\mathcal{P}}\rightarrow\mathcal{V}^{i+1}_{\mathcal{P}}$ such that $f(C_i)=C_{i+1}$where C_{i+1} is a coarsening of C_i, and for all $V_i\in\mathcal{V}^i_{\mathcal{P}}\setminus\{C_i\}$, $f(V_i)=V_i$. There is also a bijection $g{:}\mathcal{E}^i\rightarrow\mathcal{E}^{i+1}$ such that, for any $g((W,Z))=\left(W_g,Z_g\right)$: i) if $W=C_i$, then $W_g=C_{i+1}$, ii) if $Z=C_i$, then $Z_g=C_{i+1}$, iii) if $W\neq C_i$, then $W_g=W$ and iv) if $Z\neq C_i$, then $Z_g=Z$.

of causal variables are those that preserve information about the conditions under which an effect variable *E* will change, as compared to more fine-grained alternatives.

Measuring Stability Using Information Loss

Recall from our earlier discussion that we can measure the stability of a causal relationship $C \rightarrow E$ embedded in a particular causal Bayes net by removing a set of variables $\boldsymbol{B}$ from that Bayes net and assessing how much information is lost in the move from the original Bayes net to the Bayes net that is created by removing the background variables. This claim can now be made precise, using our proposed measure of information loss. Let $\mathcal{G}_{\mathcal{P}} = \left(\mathcal{V}_{\mathcal{P}}, \mathcal{E}_{\mathcal{V}_{\mathcal{P}}}\right)$ be a causal Bayes net containing a cause *C*, an effect *E*, and a set of background variables $\boldsymbol{B}$. Let $\mathcal{G}_{\mathcal{P}}^{-\boldsymbol{B}} = \left(\mathcal{V}_{\mathcal{P}}^{-\boldsymbol{B}}, \mathcal{E}_{\mathcal{V}_{\mathcal{P}}^{-\boldsymbol{B}}}\right)$ be a causal Bayes net with the same structure as $\mathcal{G}_{\mathcal{P}}$, but with all variables in $\boldsymbol{B}$ and all edges going into or out of variables in $\boldsymbol{B}$ removed.[24] The causal relationship between *C* and *E* is **stable** with respect to background condition $\boldsymbol{B}$ to the extent that the value of $\mathcal{L}\left(\mathcal{G}_{\mathcal{P}}, \mathcal{G}_{\mathcal{P}}^{-\boldsymbol{B}}, \{C\} \cup \boldsymbol{B}, \{C\}, E, q\right)$is low. That is, the relationship $C \rightarrow E$ is stable with respect to $\boldsymbol{B}$ to the extent that the average amount of information about *E* that is communicated by interventions on both *C* and the variables in $\boldsymbol{B}$ is similar to the average amount of information about *E* that is communicated solely by interventions on *C*.

[24] That is, $\mathcal{V}_{\mathcal{P}}^{-\boldsymbol{B}} = \mathcal{V}_{\mathcal{P}} \setminus \boldsymbol{B}$ and $\mathcal{E}_{\mathcal{V}_{\mathcal{P}}^{-\boldsymbol{B}}} = \{X, Y: X \in \boldsymbol{B} \vee Y \in \boldsymbol{B}\}$.